CONTENTS

0.1 Solving Degree One Equations

In this section we intend to do two important things. One is review our degree-one solving skills. Second we would like to see just a small glimpse of how some people may find some utility in these skills.

First let us review our degree-one solving skills. Recall a typical linear equation may look something like this:

$$3(x+2) = 21$$

Our preferred method involved, *free and shuffle* the variables, then *collect and kill*. The *free* part looks like this:

$$\begin{aligned} 3(x+2) &= 21 && \text{(given)} \\ 3x+6 &= 21 && \text{(DL, BI)} \end{aligned}$$

Now that we free'd the term with x we can shuffle, the x-less terms to the right, the x-terms stay on the left:

$$\begin{aligned} 3(x+2) &= 21 && \text{(given)} \\ 3x+6 &= 21 && \text{(DL, BI)} \\ 3x+6+-6 &= 21+-6 && \text{(CLA)} \\ 3x &= 15 && \text{(AiD, BI)} \end{aligned}$$

Next we collect and kill, but in this case we have only one term with x's so it is already collected, thus we need only kill the coefficient.

$$\begin{aligned} 3(x+2) &= 21 && \text{(given)} \\ 3x+6 &= 21 && \text{(DL, BI)} \\ 3x+6+-6 &= 21+-6 && \text{(CLA)} \\ 3x &= 15 && \text{(AiD, BI)} \\ x &= \frac{15}{3} && \text{(KTC)} \\ x &= 5 && \text{(Bi)} \end{aligned}$$

We have solved the linear equation, and we have reviewed our super powerful strategy *free & shuffle then collect & kill.* It might also be good to note here this is *not the only* strategy that works, it is however, *one that always works.* For further review, and more interesting problems the reader is encouraged to review the last section of chapter 3. For now, we will turn to our second objective at hand, finding utility in this.

I write the following with some very deep reservations. I other words, we include here a couple 'real-life' application problems that show how some utility in all this, yet we maintain this is only a small side benefit of your journey, not necessary for the enjoyment of algebra.

Example Suppose there is some unknown number x. Suppose we do know that 3 times the sum of the number and 7 is equal to 27. Determine the number.

Solution:

$$\begin{aligned} \text{3 times the sum of } x \text{ and 7 is equalto 27} & & \text{(given)} \\ 3(x+7) &= 27 & \text{(interpret)} \\ 3x + 21 &= 27 & \text{(DL, BI)} \\ 3x + 21 + -21 &= 27 + -21 & \text{(CLA)} \\ 3x &= 6 & \text{(BI)} \\ x &= \frac{6}{3} & \text{(KTC)} \\ x &= 2 & \text{(BI)} \end{aligned}$$

We can easily check our work: check if "3 times the sum of 2 and 7 is equal to 27" Indeed, $3 \times 9 = 27$

Example Suppose the current Celsius temperature times 9 is equal to 5 times the quantity of 32 subtracted from the corresponding temperature in Fahrenheit degrees. If the current temperature is 33° Celsius, what is the current temperature in Fahrenheit degrees?

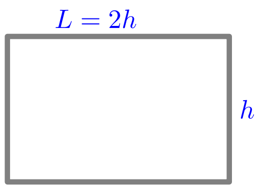
Solution:

$$
\begin{aligned}
\text{c times 9 is equal 5 times (32 subtracted from f)} & & \text{(given)}\\
9c &= 5(f-32) & \text{(interpret)}\\
9\cdot 33 &= 5(f-32) & \text{(sub c=33)}\\
297 &= 5f - 5\cdot 32 & \text{(Bi, DL)}\\
297 &= 5f - 160 & \text{(Bi)}\\
297+160 &= 5f & \text{(Bi)}\\
457 &= 5f & \text{(AT)}\\
\frac{457}{5} &= f & \text{(KTC)}\\
91.4 &= f & \text{(calculator)}
\end{aligned}
$$

Example Suppose a farmer wants to fence a rectangular piece of land using exactly 120 ft of fence, and also wants the length to be twice the height. Find the correct dimensions for the piece of land.

Solution:

A picture is often helpful.



$$
\begin{aligned}
\text{total fence is equal to 120 ft.} & & \text{(given)}\\
h+h+2h+2h &= 120ft & \text{(interpret)}\\
1\cdot h+1\cdot h+2h+2h &= 120ft & \text{(miD)}\\
(1+1+2+2)h &= 120ft & \text{(DL)}\\
h &= \frac{120ft}{1+1+2+2} & \text{(KTC)}\\
h &= \frac{120ft}{6} & \text{(KTC)}\\
h &= 20ft & \text{(BI)}
\end{aligned}
$$

Since the length is twice as much it must be $40ft$ long.

0.1.1 Exercises

1. Suppose a farmer wants to fence a rectangular piece of land using exactly 250 ft of fence, and also wants the length to be 4 times the height. Find the correct dimensions for the piece of land.

2. If the current temperature is 28° Celsius, what is the current temperature in Fahrenheit degrees? [see example, assume $9c = 5(f - 32)$]

3. Suppose a farmer wants to fence a rectangular piece of land using exactly 420 ft of fence, and also wants the length to be 5 times the height. Find the correct dimensions for the piece of land.

4. Suppose a farmer wants to fence a rectangular piece of land using exactly 360 ft of fence, and also wants the length to be 5 times the height. Find the correct dimensions for the piece of land.

5. If the current temperature is 6° Celsius, what is the current temperature in Fahrenheit degrees? [see example, assume $9c = 5(f - 32)$]

6. If the current temperature is 42° Celsius, what is the current temperature in Fahrenheit degrees? [see example, assume $9c = 5(f - 32)$]

7. If the current temperature is 17° Celsius, what is the current temperature in Fahrenheit degrees? [see example, assume $9c = 5(f - 32)$]

8. If the current temperature is 38° Celsius, what is the current temperature in Fahrenheit degrees? [see example, assume $9c = 5(f - 32)$]

9. Suppose there is some unknown number, V. Suppose we do know that 3 times the sum of V and 7 is equal to 6. Determine the value of the number, V.

10. Suppose there is some unknown number, $\star$. Suppose we do know that 3 times the sum of $\star$ and 6 is equal to -9. Determine the value of the number, $\star$.

0.2 Zero Factor Theorem

Having practiced solving *linear equations* at the end of chapter 3 and again in the last section, it is time to turn our attention to slightly more interesting equations. The natural direction of our progress is to solve degree two equations, often called *quadratic equations*. Naturally, after we become good at degree two equations, we want to explore degree 3 and higher. As it turns out, the key ingredient for all of these, is something we call the theorem:

Theorem 0.2.1. *The Zero Factor Theorem [ZFT] say if A and B are numbers or symbols for which our axioms apply and if*

$$A \cdot B = 0$$

then

$$A = 0 \qquad OR \qquad B = 0$$

The following may be a helpful analogy. If the product $A \cdot B = 0$, then somehow they killed each other in the act of multiplying by each other. Roughly speaking, the ZFT says this *can not* happen *unless* one of the two was the killer, either $A = 0$ or $B = 0$

This amazingly simply little fact, turns out to be the key to solving so many if not all of our higher degree equations, as we shall soon see. For now, let us be content with just practicing the ZFT.

Example Suppose

$$x(x+3) = 0$$

then by ZFT

$$x = 0 \qquad \text{OR} \qquad x + 3 = 0$$

Example Suppose

$$(x-5)(x+3) = 0$$

then by ZFT

$$x - 5 = 0 \qquad \text{OR} \qquad x + 3 = 0$$

Example It even works for the product of several items. Suppose

$$(2x-5)(3x^2+1)(x^3+x) = 0$$

then by ZFT

$$2x - 5 = 0 \qquad \text{OR} \qquad 3x^3 + 1 = 0 \qquad \text{OR} \qquad x^3 + x = 0$$

Example it works on any product, so long as the numbers obey our axioms:
Suppose

$$5 \cdot x \cdot (x^2 + 1) = 0$$

then by ZFT

$$5 = 0 \qquad \text{OR} \qquad x = 0 \qquad \text{OR} \qquad x^2 + 1 = 0$$

Example Suppose

$$\left(\sqrt{x}\right)\left(1 + \frac{1}{x}\right) = 0$$

then by ZFT

$$\sqrt{x} = 0 \qquad \text{OR} \qquad 1 + \frac{1}{x}$$

The big advantage is that often we manage to break down a problem using ZFT into little problems which we can finish, for example if we can break them into little degree one problems. Such as the example below.

Example Suppose

$$(2x - 5)(3x + 1) = 0$$

then by ZFT

$$2x - 5 = 0 \qquad \text{OR} \qquad 3x + 1 = 0$$

then by CLA & BI

$$2x = 5 \qquad \text{OR} \qquad 3x = -1$$

finally, BI

$$x = 5/2 \qquad \text{OR} \qquad x = -1/3$$

and we have solved the equation.

Of course the challenge will be when the equation is not given in the factored form as it was above, $(2x-50)(2x+12) = 0$. Suppose it was given in its expanded form, as if someone had gone through it and expanded [foiled] the terms and one had to solve

$$6x^2 - 13x - 5 = 0$$

At first sight this might appear monstrous. Imagine someone asked you, "got an unknown number x, the hint is 'x square times 6 minus the number x times 13, from that subtract 5, the result is 0, *guess what the number is!*" That is no enchiladas [no walk in the park], unless of course you see the power of the ZFT, once we factor it the entire process looks like this..

$$\begin{aligned} 6x^2 - 13x - 5 &= 0 && \text{(given)} \\ 6x^2 - 15x + 2x - 5 &= 0 && \text{(BI, the split)} \\ 3x(2x-5) + 1(2x-5) &= 0 && \text{(Dl, BI)} \\ (3x+1)(2x-5) &= 0 && \text{(Dl)} \\ 3x+1=0 \qquad \text{OR} &\qquad 2x-5=0 && \text{(ZFT)} \\ x=-1/3 \qquad \text{OR} &\qquad x=5/2 && \text{(BI)} \end{aligned}$$

And we have solved our first degree two equation. Note the importance of our factoring skills is eminent and can not be overemphasized. We must be masters of factoring, otherwise, none of this will go down this smoothly. One could say that once you have factored the equation with factor on one side and zero on the other side, you may want to throw out a celebratory "winner winner chicken dinner" remark under your breath, because the hard part is complete. Another example.. last one..

Example

$$\begin{aligned} x^2 + 3x - 4 &= 0 && \text{(given)} \\ x^2 + 4x + -1x + -4 &= 0 && \text{(BI)} \\ x(x+4) + -1(x+4) &= 0 && \text{(DL, BI)} \\ (x-1)(x+4) &= 0 && \text{(DL)} \\ & && \text{(ready for ZFT, winner winner..)} \\ x-1=0 \quad \text{OR} &\quad x+4=0 && \text{(ZFT)} \\ x=1 \quad \text{OR} &\quad x=-4 && \text{(BI)} \end{aligned}$$

At this point there are two important tasks at hand, to practice solving equations by ZFT, and to prove ZFT, both at left for the reader to enjoy.

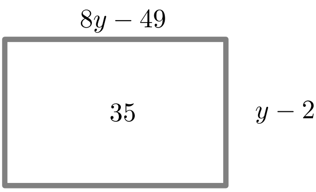
0.2.1 Exercises

1. Suppose there is a secret number in a box, call it q. Suppose 28 plus the quantity -6 times the number is the same as the number times the quantity of -9 plus the number. Determine the value of the number q.

2. Solve for w:

$$6w^2 - 2w - 9 = 7w - 3$$

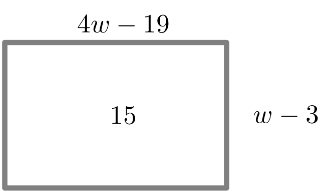
3. Suppose a rectangle has area $35sqft$ and the dimensions of the rectangle are as given below. Determine the size of each of the sides of the rectangle.



4. Solve for t:

$$t^2 + 41t + 29 = 29t - 6$$

5. Suppose a rectangle has area $15sqft$ and the dimensions of the rectangle are as given below. Determine the size of each of the sides of the rectangle.



6. Suppose there is a secret number in a box, call it u. Suppose -14 plus the quantity 8 times the number is the same as the number times the quantity of -1 plus the number. Determine the value of the number u.

7. Solve for r:

$$6r^2 + 22r + 12 = 0$$

8. Suppose there is a secret number in a box, call it v. Suppose 48 plus the quantity -3 times the number is the same as the number times the quantity of -1 plus the number. Determine the value of the number v.

9. Solve for x:

$$6x^2 + x + -15 = 0$$

10. Solve for q:

$$3q^2 + 0 + -12 = 0$$

11. Suppose there is a secret number in a box, call it r. Suppose -27 plus the quantity -8 times the number is the same as the number times the quantity of -20 plus the number. Determine the value of the number r.

12. Solve for z:

$$z^2 + 4z + -21 = 0$$

13. Solve for q:

$$2q^2 + 16q - 23 = 19q - 3$$

14. Solve for u:

$$9u^2 - 10u + 4 = 11u - 6$$

15. Solve for v:

$$6v^2 + 17v - 1 = 7v + 3$$

16. Solve for q:

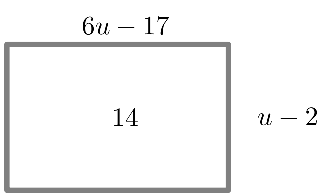
$$3q^2 + 35q + 7 = 23q - 5$$

17. Solve for v:

$$4v^2 + 9v - 13 = 5v - 5$$

18. Suppose there is a secret number in a box, call it r. Suppose 9 plus the quantity -2 times the number is the same as the number times the quantity of -10 plus the number. Determine the value of the number r.

19. Suppose a rectangle has area $14 sqft$ and the dimensions of the rectangle are as given below. Determine the size of each of the sides of the rectangle.



20. Solve for y:

$$4y^2 + 2y + -6 = 0$$

21. Solve for z:

$$9z^2 - 12z - 11 = 3z + 3$$

22. Solve for t:

$$3t^2 + 22t + 7 = 0$$

23. Solve for x:

$$x^2 + -2x + -35 = 0$$

24. Solve for t:

$$6t^2 - 4t - 47 = 5t - 5$$

25. Solve for q:

$$3q^2 + -5q + -28 = 0$$

0.3 SRP & Roots

We continue our journey in solving polynomial equations. To continue, we will need to develop a couple more vocabulary words. Here we define what we mean when we say *roots*.

Definition Suppose a is a number for which our axioms apply. Then, the solution/s to the equation

$$x^2 = a$$

are called the *square roots* of a.

Example Determine the roots of 9.

Solution:

we need only solve

$$x^2 = 9$$

since $x = 3$ and $x = -3$ each solve the equation, these are called the square roots of 9.

Example Determine the roots of 25.

Solution:

we need only solve

$$x^2 = 25$$

since $x = 5$ and $x = -5$ each solve the equation, these are called the square roots of 25.

Most number we encounter in our class will have two square roots. If one is a positive root and the other is negative we make a distinction and call the positive one the *principal square root*, and we even make a symbol for it.

Definition *def of radical [def of $\sqrt{}$]* Suppose a is a positive number for which our axioms apply. Then, the *positive* solution to the equation

$$x^2 = a$$

is called the *principal square root* of a and we denote it as such

$$\sqrt{a} = \text{the principal square root of } a$$

.

Example determine $\sqrt{25}$

Solution:

since 5 is the positive solution to $x^2 = 25$ then by def of $\sqrt{\quad}$

$$\sqrt{25} = 5$$

Example determine $\sqrt{36}$

Solution:

since 6 is the positive solution to $x^2 = 36$ then by def of $\sqrt{\quad}$

$$\sqrt{36} = 6$$

Example determine $\sqrt{\frac{4}{9}}$

Solution:

since $\frac{2}{3}$ is the positive solution to $x^2 = \frac{4}{9}$ then by def of $\sqrt{\quad}$

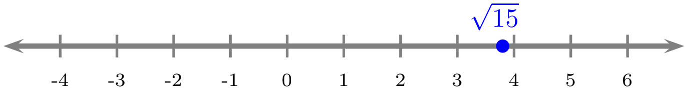
$$\sqrt{\frac{4}{9}} = \frac{2}{3}$$

Sometimes, it is not immediately clear to determine exactly what a square root is equal to. For example consider the following.

Example determine $\sqrt{15}$

Solution:

we need to find the positive solution to $x^2 = 15$ It appears none of our integers solves this, nor our rational numbers. The number 4 almost works but $4^2 = 16$, so 4 seems a bit too much, while 3 is not enough since $3^2 = 9$, yet it is not difficult to imagine that some number close to 4 but slightly less than 4 works. Even without knowing all the digits of the exact number we can 'name' beast and picture it's place in some sort of number line. We shall name this number naturally, $\sqrt{15}$, and picture it somewhere between 3 and 4. Perhaps like this:



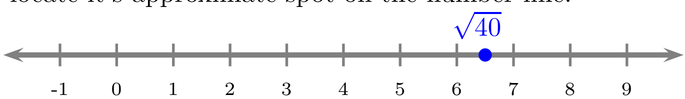
It is important to note here that the above number line contains, all natural numbers, $\mathbb{N}$, contains all whole number, integers, rationals, and then lots more number in between the rationals, such as $\sqrt{15}$. The collection of all of these number is called the *real numbers* for which we use the symbol $\mathbb{R}$ Numbers such as $\sqrt{15}$ which are real but not rational are called *irrational numbers*.

Here is another example of an irrational number.

Example determine $\sqrt{40}$

Solution:

we need to find the positive solution to $x^2 = 40$ yet, $6^2 = 36$ but $7^2 = 49$, so the solutions lies somewhere between 6 and 7, but the decimal representation is too long, endless in fact, so we just represent it is as $\sqrt{40}$. We can however locate it's approximate spot on the number line.



These ideas are much bigger than the examples as they can easily be generalized. For example, we conceive of not just square roots, but *cubic roots*. Consider the following analogous.

Another example,

Definition Suppose $a \in \mathbb{R}$. Then, the solution/s to the equation

$$x^3 = a$$

are called the *cubic roots* of a.

Definition Suppose $a \in \mathbb{R}$. If there is a *real number solution* to the equation

$$x^3 = a$$

such solution is called a *principal cubic root* of a. and we define

$$\sqrt[3]{a} = \text{the principla cubic root of } a$$

Example determine $\sqrt[3]{8}$

Solution:

We need only find the real solution[1] to

$$x^3 = 8$$

Since, $x = 2$ works and is real then

$$\sqrt[3]{8} = 2$$

Example determine $\sqrt[3]{27}$

Solution:

We need only find the real solution to

$$x^3 = 27$$

Since, $x = 3$ works and is real then

$$\sqrt[3]{27} = 3$$

Example determine $\sqrt[3]{125}$

Solution:

We need only find the real solution to

$$x^3 = 125$$

Since, $x = 5$ works and is real then

$$\sqrt[3]{125} = 5$$

Example determine $\sqrt[3]{\frac{-1}{64}}$

Solution:

We need only find the real solution to

$$x^3 = \frac{-1}{64}$$

Since, $x = \frac{-1}{4}$ works and is real then

$$\sqrt[3]{\frac{-1}{64}} = \frac{-1}{4}$$

[1] it takes some work to show only one real solution is possible

Finally, for this section, we need to tie all these ideas together. The conclusion is that now we have developed the ideas and notations to swiftly solve equations of the type

$$x^2 = a$$

As follows.

$$\begin{aligned}
x^2 &= a && \text{(given)}\\
x^2 - a &= 0 && \text{(CLA, Bi)}\\
x^2 - \left(\sqrt{a}\right)^2 &= 0 && \text{(def of } \sqrt{\ }\text{)}\\
\left(x - \sqrt{a}\right)\left(x + \sqrt{a}\right) &= 0 && \text{(DS)}\\
x - \sqrt{a} = 0 \qquad \text{OR} \qquad x + \sqrt{a} &= 0 && \text{(ZFT)}\\
x = \sqrt{a} = 0 \qquad \text{OR} \qquad x &= -\sqrt{a} && \text{(BI)}
\end{aligned}$$

We often summarize the last step as

$$x = \pm\sqrt{a}$$

We also package the entire sequence of steps into theorem that will soon become famous for us.

Theorem 0.3.1. *The square root property [SRP] If a is a number for which our axioms apply and*

$$x^2 = a$$

then

$$x = \pm\sqrt{a}$$

In practice this is how may use the SRP

Example Solve $x^2 = 7$

Solution:

$$\begin{aligned}
x^2 &= 7 && \text{(given)}\\
x &= \pm\sqrt{7} && \text{(SRP)}
\end{aligned}$$

Example Solve $y^2 = 12$

Solution:

$$\begin{aligned} y^2 &= 12 && \text{(given)} \\ y &= \pm\sqrt{12} && \text{(SRP)} \end{aligned}$$

Example Solve $5t^2 = 3$

Solution:

$$\begin{aligned} 5t^2 &= 3 && \text{(given)} \\ t^2 &= \frac{3}{5} && \text{(given)} \\ t &= \pm\sqrt{\frac{3}{5}} && \text{(SRP)} \end{aligned}$$

Example Solve $2t^2 = 10 + \pi$

Solution:

$$\begin{aligned} 2t^2 &= 10 + \pi && \text{(given)} \\ t^2 &= \frac{10 + \pi}{2} && \text{(given)} \\ t &= \pm\sqrt{\frac{10 + \pi}{2}} && \text{(SRP)} \end{aligned}$$

Example Solve $(blah)^2 = 10$

Solution:

$$
\begin{aligned}
(blah)^2 &= 10 && \text{(given)}\\
blah &= \pm\sqrt{10} && \text{(SRP)}
\end{aligned}
$$

Example Solve $(x-3)^2 = 10$

Solution:

$$
\begin{aligned}
(x-3)^2 &= 10 && \text{(given)}\\
x-3 &= \pm\sqrt{10} && \text{(SRP)}\\
x &= 3 \pm \sqrt{10} && \text{(CLA, Bi)}
\end{aligned}
$$

0.3.1 Exercises

1. Use a calculator to approximate:

$$\sqrt[3]{-42}$$

2. Use a calculator to approximate:

$$\sqrt{\frac{4}{3}}$$

3. Solve for $12t+4$ in:

$$(12t+4)^2 = \frac{16}{9}$$

4. Compute

$$\sqrt[3]{-\frac{64}{125}}$$

5. Solve for $t+14$ in:

$$(t+14)^2 = \frac{9}{16}$$

6. Solve for $y+13$ in:

$$(y+13)^2 = 9$$

7. Compute

$$\sqrt{49}$$

8. Compute

$$\sqrt[3]{-1}$$

9. Solve for $12v+2$ in:

$$(12v+2)^2 = 1$$

10. Solve for v in:

$$9(v)^2 = \frac{25}{4}$$

11. Solve for W in:

$$W^2 = 64$$

12. Use a calculator to approximate:

$$\sqrt{\frac{4}{5}}$$

13. Solve for $v + 8$ in:

$$(v + 8)^2 = 16$$

14. Use a calculator to approximate:

$$\sqrt{35}$$

15. Use a calculator to approximate:

$$\sqrt[3]{-44}$$

16. Solve for $z + 7$ in:

$$(z + 7)^2 = 169$$

17. Solve for $3u + 12$ in:

$$(3u + 12)^2 = \frac{4}{25}$$

18. Compute

$$\sqrt[3]{\frac{8}{125}}$$

19. Compute

$$\sqrt{64}$$

20. Solve for $2x + 7$ in:

$$(2x + 7)^2 = 4$$

0.4 More Roots & Introduction of i

Time for a small summary of the our ever expanding collection of numbers. In the beginning, we started with the natural numbers:

$$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$$

We one way to see our progress is to to recall how we came about our next number. We asked ourselves if there was a 'water-like' element that left everything unchanged when added to it. Now, having solved a few equation along our journey, we could rephrase that very same question in the language of equations. Instead of asking for example ”is there a number that does nothing to 3 when added to it?” we can ask ”is there a number x such

$$x + 3 = 3$$

No such number existed in our $\mathbb{N}$ limited world at that point, thus we *invented* a new number 0 to solve precisely this type of equation, $x + 3 = 3$, with such invention, our number world grew a little and we declared our new world to be the world of whole number

$$\mathbf{W} = \{0, 1, 2, 3, 4, \ldots\}$$

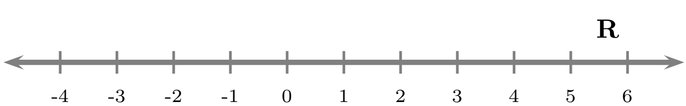
Similarly, we could rephrase the existence question of additive inverse. Instead of asking if there is an additive killer of 3, we can ask is there a solution to

$$x + 3 = 0$$

The **W** world contained no solutions to such equation, thus we *invented* negative numbers and our world of numbers grew again to include such new numbers

$$\mathbf{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

We continued our pattern of inventing numbers to solve equation that have no solutions, and our world of numbers grew to include the rationals, **Q**, the irrationals, then the reals, **R**. Bringing us to our current state, where the collection of all numbers we know is contained in this, the real number line.



Now, our world is about to grow again. This time we are looking for a number that when multiplied times itself yields -1. Our current world is limited to real numbers, all of which are negative, 0, or positive. Surely none of the negative numbers would work because of the NNT, we know a negative times a negative would be a positive, so it would not be equal to -1. Similarly, none of the positive numbers could possibly solve

$$x^2 = -1$$

.. and so it is time again to celebrate the inventiveness and indomitable human spirit. Since no real number solves this, we invent a new number, and call it i for imaginary number and just like that i solves the equation as such:

$$i^2 = -1$$

With the invent of our new jewel, i, comes and avalanche of questions. Such as what is $i + 3$, or $i + i$, or i^5, and what do we call these new numbers, and where do these numbers live? etc. etc. As it turns out, these numbers will play nice with all of our axioms, with the new added detail, that every i^2 can be exchanged for a -1 since $i^2 = -1$, Said differently, by *definition of i [def i]*,

$$i = \sqrt{-1}$$

In due time, we may answer all these questions for now, we turn our attention to the task at hand, roots and solving equations. We proceed with a new and important theorem.

Theorem 0.4.1. *Product of Radicals is Radical of Products [RP=PR] If a and b are real numbers, and at least one of them is a positive number, then:*

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

Here is one way we can use this theorem, for example to simplify [or 're-write'] radicals.

Example Simplify $\sqrt{12}$

$$\begin{aligned} \sqrt{12} &= \sqrt{4 \cdot 3} && \text{(TT)} \\ &= \sqrt{4} \cdot \sqrt{3} && \text{(RP=PR)} \\ &= 2\sqrt{3} && (\text{def } \sqrt{}) \end{aligned}$$

Example Simplify $\sqrt{108}$

$$\begin{aligned}
\sqrt{108} &= \sqrt{4 \cdot 9 \cdot 3} && \text{(TT)} \\
&= \sqrt{4} \cdot \sqrt{9} \cdot \sqrt{3} && \text{(RP=PR)} \\
&= 2 \cdot 3 \cdot \sqrt{3} && \text{(def } \sqrt{}\text{)} \\
&= 6\sqrt{3} && \text{(BI)}
\end{aligned}$$

Also with this theorem, we can now handle negative radicals.

Example Simplify $\sqrt{-5}$

$$\begin{aligned}
\sqrt{-5} &= \sqrt{-1 \cdot 5} && \text{(NPT)} \\
&= \sqrt{-1} \cdot \sqrt{5} && \text{(RP=PR)} \\
&= i\sqrt{5} && \text{(def } i\text{)}
\end{aligned}$$

We collect the steps from the previous example, generalize and pack them into a theorem.

Theorem 0.4.2. *Negative Radicals [nRad] If a is a real number, then:*

$$\sqrt{-a} = i\sqrt{a}$$

We could use it, again to simplify negatve radicals, such as:

Example Simplify $\sqrt{-17}$

$$\sqrt{-17} = i\sqrt{17} \qquad \text{(nRad)}$$

Example Simplify $\sqrt{-17}$

$$\begin{aligned}
\sqrt{-12} &= i\sqrt{12} && \text{(nRad)} \\
&= i\sqrt{4 \cdot 3} && \text{(TT)} \\
&= i\sqrt{4} \cdot \sqrt{3} && \text{(RP=PR)} \\
&= i \cdot 2 \cdot \sqrt{3} && \text{(def)} \\
&= 2i\sqrt{3} && \text{(BI)}
\end{aligned}$$

Having said that, we can now solve equations such as

$$x^2 = -5$$

Example Solve $x^2 = -5$

$$\begin{aligned} x^2 &= -5 && \text{(given)} \\ x &= \pm\sqrt{-5} && \text{(SRP)} \\ x &= \pm i\sqrt{5} && \text{(nRad)} \end{aligned}$$

Example Solve $(x+3)^2 = -7$

$$\begin{aligned} (x+3)^2 &= -7 && \text{(given)} \\ x+3 &= \pm\sqrt{-7} && \text{(SRP)} \\ x+3 &= \pm i\sqrt{7} && \text{(nRad)} \\ x &= -3 \pm i\sqrt{7} && \text{(CLA, Bi)} \end{aligned}$$

0.4.1 Exercises

1. Solve for q in:

$$q^2 = -\frac{4}{3}$$

2. Simplify:

$$(i-4)(3i-6)$$

3. Compute

$$\sqrt{-15}$$

4. Compute

$$\sqrt{-3}$$

5. Simplify:

$$(i-7)(3i-5)$$

6. Solve for q in:

$$q^2 = -\frac{2}{5}$$

7. Solve for t in:

$$(t-14)^2 = -14$$

8. Solve for q in:

$$(q-8)^2 = -8$$

9. Solve for v in:

$$v^2 = -\frac{4}{5}$$

10. Solve for z in:

$$(z-15)^2=-2$$

11. Compute

$$\sqrt{-2}$$

12. Solve for v in:

$$v^2=-1$$

13. Solve for y in:

$$(y-15)^2=-\frac{3}{2}$$

14. Compute

$$\sqrt{-7}$$

15. Solve for w in:

$$w^2=-\frac{5}{3}$$

16. Compute

$$\sqrt{-15}$$

17. Solve for w in:

$$w^2=-\frac{4}{3}$$

18. Solve for z in:

$$(z-14)^2=-1$$

19. Solve for r in:

$$(r-7)^2=-5$$

20. Compute

$$\sqrt{-\frac{4}{5}}$$

0.5 Solve by PP2

Let us take a moment to summarize some of the highlights of this chapter thus far. We have learned about the Zero Factor Theorem, which helps us solve equations so long as we can factor them as in

$$x^2 + 3x + 2 = 0$$

becomes

$$(x + 2)(x + 1) = 0$$

which we break up into smaller equations using ZFT.

In addition, we have learned to solve equations where the term with the sought variable is a perfect square as in

$$(x + 3)^2 = 10$$

Using mainly SRP this equation becomes

$$x = -3 \pm \sqrt{10}$$

In this way we are making progress towards being able to solve all degree two equations. That is our goal at the moment, we want to be able to solve *all quadratic equations.*

This brings us to our next installment of skills. Here we consider quadratic equations such as

$$x^2 + 6x + 2 = 0$$

which is neither a perfect square nor is it one we can factor using our current factoring skills. Right about now would a great time to use our 3 lb. brain and come up with something amazingly creative to solve this equation. Indeed, that is what we do. The answer lies on a little polynomial we've already met. The polynomial is *Pascal Polynomial #2*. It turns out this idea will nearly complete our degree story, as *every degree two equation can be solved* using the PP2 strategy we are about to present. That is how important this idea is, it will solve every quadratic equation under the sun. We will practice it here, and turn it into a template in the next section.

At last here is the strategy. roughly speaking it involves turning polynomials into a PP2 then using SRP to solve them. Since we are already very good at using SRP, this will flow nicely. Recall the PP2 pattern

$$X^2 + 2YX + Y^2 = (X + Y)^2$$

Now...

Example Solve $x^2 + 6x + 2 = 0$

Solution:

$$
\begin{aligned}
x^2 + 6x + 2 &= 0 && \text{(given)} \\
x^2 + 6x &= -2 && \text{(CLA, BI)} \\
x^2 + 2 \cdot 3x &= -2 && \text{(TT)}
\end{aligned}
$$

and the left side is already looking like a PP2, matching the first terms $X^2 + 2YX + Y^2$ where the $Y = 3$. There is a missing item here, it is a Y^2 or in our case a 3^2 we can legally add it to both sides completing the perfect square, or completing the PP2 pattern.

$$
\begin{aligned}
x^2 + 6x + 2 &= 0 && \text{(given)} \\
x^2 + 6x &= -2 && \text{(CLA, BI)} \\
x^2 + 2 \cdot 3x &= -2 && \text{(TT)} \\
x^2 + 2 \cdot 3x + 3^2 &= -2 + 3^2 && \text{(CLA, BI)} \\
(x+3)^2 &= -2 + 3^2 && \text{(PP2)} \\
(x+3)^2 &= 7 && \text{(BI)}
\end{aligned}
$$

Now it smells like victory, by completing the PP2 we have turned into a problem where we can use SRP as we practiced so diligently on the previous section. SRP will take us home on this one..

$$\begin{aligned}
x^2+6x+2&=0 && \text{(given)}\\
x^2+6x&=-2 && \text{(CLA, BI)}\\
{\color{blue}x^2+2\cdot 3x}&=-2 && \text{(TT)}\\
x^2+2\cdot 3x+3^2&=-2+3^2 && \text{(CLA, BI)}\\
(x+3)^2&=-2+3^2 && \text{(PP2)}\\
(x+3)^2&=7 && \text{(BI)}\\
x+3&=\pm\sqrt{7} && \text{(SRP)}\\
x&=-3\pm\sqrt{7} && \text{(CLA, BI)}
\end{aligned}$$

Yipie Kae Yeaeaee!!!

Such is the strategy, and it will solve all degree two equations. The strategy is sometimes called *completing the square* or *completing PP2*

Example Solve $x^2+14x=5$

Solution:

$$\begin{aligned}
x^2+14x&=5 && \text{(given)}\\
{\color{blue}x^2+2\cdot 7x}&=5 && \text{(TT)}\\
x^2+2\cdot 7x+7^2&=5+7^2 && \text{(CLA, BI)}\\
(x+7)^2&=5+7^2 && \text{(PP2)}\\
(x+7)^2&=54 && \text{(BI)}\\
x+7&=\pm\sqrt{54} && \text{(SRP)}\\
x+7&=\pm 3\sqrt{6} && \text{(BI)}\\
x&=-7\pm 3\sqrt{6} && \text{(CLA, BI)}
\end{aligned}$$

The next example shows that any real number can be written as a product of 2 times something. This was and is useful in matching the middle term of the PP2 since it has a $2YX$ in it. In the previous couple examples we wrote $14 = 2 \cdot 7$ and $6 = 2 \cdot 3$ but really anything can be written as 2 times something, consider

$$5 = 2 \cdot \frac{5}{2} \qquad \text{OR} \qquad 3 = 2 \cdot \frac{3}{2} \qquad \text{OR} \qquad 15 = 2 \cdot \frac{15}{2}$$

Example Solve $x^2 + 5x = 2$

Solution:

$$\begin{aligned}
x^2 + 5x &= 2 && \text{(given)} \\
x^2 + 1 \cdot 5x &= 2 && \text{(Mid)} \\
x^2 + \left(2 \cdot \frac{1}{2}\right) \cdot 5x &= 2 && \text{(MinV)} \\
x^2 + 2\left(\frac{1}{2} \cdot 5\right) x &= 2 && \text{(ALM)} \\
x^2 + 2 \cdot \frac{5}{2} \cdot x &= 2 && \text{(BI)} \\
x^2 + 2 \cdot \frac{5}{2} \cdot x + \left(\frac{5}{2}\right)^2 &= 2 + \left(\frac{5}{2}\right)^2 && \text{(CLA)} \\
\left(x + \frac{5}{2}\right)^2 &= 8/4 + 25/4 = 33/4 && \text{(PP2, BI)} \\
x + \frac{5}{2} &= \pm\sqrt{33/4} && \text{(SRP)} \\
x &= -\frac{5}{2} \pm \sqrt{33/4} && \text{(CLA, BI)}
\end{aligned}$$

0.5.1 Exercises

1. Solve by completing the square [complete the PP2]:

$$q^2 + q = -2$$

2. Solve by completing the square [complete the PP2]:

$$y^2 + y = 6$$

3. Solve by completing the square [complete the PP2]:

$$x^2 - 6x = 7$$

4. Solve by completing the square [complete the PP2]:

$$z^2 + 7z = 4$$

5. Solve by completing the square [complete the PP2]:

$$w^2 - 3w = 4$$

6. Solve by completing the square [complete the PP2]:

$$w^2 - 3w = 5$$

7. Solve by completing the square [complete the PP2]:

$$t^2 + 2t = -3$$

8. Solve by completing the square [complete the PP2]:

$$w^2 - 2w + 4 = -w - 2$$

9. Solve by completing the square [complete the PP2]:

$$r^2 - 12r - 6 = 1 - 6r$$

10. Solve by completing the square [complete the PP2]:

$$u^2 - 8u - 4 = 3 - 4u$$

11. Solve by completing the square [complete the PP2]:

$$t^2 + 10t - 3 = 5t + 1$$

12. Solve by completing the square [complete the PP2]:

$$27v - 3v^2 = -6$$

13. Solve by completing the square [complete the PP2]:

$$3u^2 - 18u = -24$$

14. Solve by completing the square [complete the PP2]:

$$-3q^2 - 30q = -15$$

15. Solve by completing the square [complete the PP2]:

$$-3t^2 - 24t = -30$$

0.6 Quadratic Formula

In this section we put a graceful end to our battle against quadratic equations. In the last section, we used the idea of completing the square to turn every possible quadratic expression into a perfect square where we could use SRP to solve the corresponding quadratic equation. In this section, we repeat the very same method, but on a template, that is on a generic equation with generic possible coefficients such as

$$ax^2 + bc + c = 0$$

The idea is, if we can solve the generic template, at once we will have solved all possible equations that can be written as the template. That is precisely what we do.

Theorem 0.6.1. *The Quadratic Formula [QF] if a, b, c are real numbers, and $a > 0$, then the solutions to*

$$ax^2 + bx + c = 0$$

are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof.

$$ax^2 + bx + c = 0 \qquad \text{(given)}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = \frac{1}{a} \cdot 0 \qquad \text{(CLM, BI)}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{(0MT)}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{(CLA, BI)}$$

$$x^2 + 1 \cdot \frac{b}{a}x = -\frac{c}{a} \qquad \text{(MiD)}$$

$$x^2 + 2 \cdot \frac{1}{2} \cdot \frac{b}{a}x = -\frac{c}{a} \qquad \text{(MinV)}$$

$$x^2 + 2 \cdot \frac{b}{2a}x = -\frac{c}{a} \qquad \text{(ALM, BI)}$$

$$x^2 + 2 \cdot \frac{b}{2a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \qquad \text{(CLA)}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \qquad \text{(PP2, BI)}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

(BI (to get common denominator))

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{(ATT, NWW, BI)}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{(SRP)}$$

$$x = \frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{(CLA, BI)}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} \qquad \text{(RQ=QR)}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{(def } \sqrt{}\text{)}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{(ATT)}$$

Example solve $2x^2 + 3x + 1 = 0$

Solution:

$$\begin{aligned}
2x^2 + 3x + 1 &= 0 && \text{(Given)}\\
x &= \frac{-3 \pm \sqrt{3^2 - 4\cdot 2\cdot 1}}{2\cdot 2} && \text{(QF)}\\
x &= \frac{-3 \pm \sqrt{9-8}}{4} && \text{(BI)}\\
x &= \frac{-3 \pm \sqrt{1}}{4} && \text{(BI)}\\
x &= \frac{-3 \pm 1}{4} && \text{(def } \sqrt{}\text{)}\\
x &= \frac{-3 + 1}{4} \qquad \text{OR} \qquad x = \frac{-3-1}{4} && \text{(def } \pm\text{)}\\
x &= \frac{-2}{4} \qquad \text{OR} \qquad x = \frac{-4}{4} && \text{(BI)}\\
x &= \frac{-1}{2} \qquad \text{OR} \qquad x = -1 && \text{(BI)}
\end{aligned}$$

Example solve

$$2x^2 + 3x + 4 = 0$$

$$\begin{aligned}
2x^2 + 3x + 4 &= 0 && \text{(Given)}\\
x &= \frac{-3 \pm \sqrt{3^2 - 4\cdot 2\cdot 4}}{2\cdot 2} && \text{(QF)}\\
x &= \frac{-3 \pm \sqrt{9-32}}{4} && \text{(BI)}\\
x &= \frac{-3 \pm \sqrt{-23}}{4} && \text{(BI)}
\end{aligned}$$

Example

$$\begin{aligned}
x^2 + 5x &= 2 && \text{(given)}\\
x^2 + 5x - 2 &= 0 && \text{(CLA, BI)}\\
x &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1(-2)}}{2 \cdot 1} && \text{(QF)}\\
x &= \frac{-5 \pm \sqrt{33}}{2} && \text{(BI)}
\end{aligned}$$

0.6.1 Exercises

1. Solve by QF:

$$3z^2 + 6z - 5 = 0$$

2. Solve by QF:

$$3r^2 + 7r + 3 = 0$$

3. Solve by QF:

$$2r^2 + 4r - 7 = 0$$

4. Solve by QF:

$$r^2 + 5r - 1 = 0$$

5. Solve by QF:

$$2y^2 - 6y + 7 = 0$$

6. Solve by QF:

$$2u^2 - 2u + 5 = 0$$

7. Solve by QF:

$$2y^2 + 5y - 3 = 0$$

8. Solve by QF:

$$3v^2 - 4v - 2 = 0$$

9. Solve by QF:

$$2w^2 + 5w - 2 = 0$$

10. Solve by QF:

$$2r^2 - 5r + 2 = 0$$

0.7 Other Equations

Having finished the degree two story, we move on to sample other type of equations. Here we consider two types of equations that are new to us.

The first type of equations we consider are ones with absolute values on them. Let us first define what we mean when we say absolute value.

Definition If a is a real number, we define the *absolute value of* a *[def absV]* to be simply a if a is already positive and we define it as $-a$ if a is negative. Said differently,

$$|a| = \begin{cases} a, & \text{if } a \text{ is positive or zero} \\ -a, & \text{if is negative} \end{cases}$$

Notice in the case where a is negative then $-a$ is the negative of a negative number thus positive, as in

Example compute $|-5|$

Solution:

$$|-5| = -(-5) \quad \text{since -5 is negative by [def absV]}$$

Often we will include NNT and other ideas from Chapter 1 into this step and simply write

$$|-5| = 5 \quad \text{by [def absV]}$$

The idea that $|x|$ is either simply x or maybe $-x$ is simple but potent enough to help us solve equations with absolute values. We summarize this into a theorem, one that should be a joy for the student to prove.

Theorem 0.7.1. *Absolute Value Theorem [AVT] if* x&a *are real numbers and*

$$|x| = a$$

then

$$x = a \qquad OR \qquad x = -a$$

Example solve $|x| = 7$

Solution:

$$
\begin{aligned}
|x| &= 7 && \text{(given)} \\
x = 7 \qquad \text{OR} \qquad x &= -7 && \text{(absV)}
\end{aligned}
$$

Example solve $|2x-3|=9$

Solution:

$$
\begin{aligned}
|2x-3| &= 9 && \text{(given)} \\
2x-3=9 \qquad \text{OR} \qquad 2x-3 &= -9 && \text{(absV)} \\
2x=12 \qquad \text{OR} \qquad 2x &= -6 && \text{(CLA, Bi)} \\
x=6 \qquad \text{OR} \qquad x &= -3 && \text{(KTC, Bi)}
\end{aligned}
$$

Next, we turn our attention to other types of equations. These involve more than one unknown quantity. Consider the following scenario. Suppose we consider two unknown quantities, x and y. Moreover, suppose we know a couple hints about them, one that $x+y=5$ and the other that $3x+y=11$. We set out as our goal to solve for these unknown quantities. We also refer to these two equations a *system of equations*. The method we will now introduce to solve these is called the method of substitution. Roughly speaking, in involves solving for one of the variables in one of the equations and substituting it onto the other equation.

Example Solve

$$
\begin{aligned}
x+y &= 5 && \text{(given: equation A)} \\
3x+y &= 11 && \text{(given: equation B)}
\end{aligned}
$$

Solution:

We take equation A and solve for x

$$
\begin{aligned}
x+y &= 5 && \text{(given: equation A)} \\
x &= -y+5 && \text{(CLA, Bi)}
\end{aligned}
$$

we now substitute this value into equation B

$$
\begin{aligned}
3x + y &= 11 && \text{(given: equation B)}\\
3(-y+5) + y &= 11 && \text{(substitute)}\\
-3y + 15 + y &= 11 && \text{(DL, BI)}\\
-2y &= 11 + -15 && \text{(BI)}\\
-2y &= -4 && \text{(BI)}\\
y &= 2 && \text{(KTC, BI)}
\end{aligned}
$$

Once we have the value of y, we substitute into either equation to obtain the value of x.

$$
\begin{aligned}
x + y &= 5 && \text{(given: equation A)}\\
x + 2 &= 5 && \text{(substitute)}\\
x &= 3 && \text{(Bi)}
\end{aligned}
$$

Thus the conclusion is $x = 3$ & $y = 2$ which can easily be verified as correct solutions to the riddle $x + y = 5$ and $3x + y = 11$.

There will be an opportunity to practice these much more extensively in the homework. For the moment we turn our attention to our last variation for this section. We now consider solving *rational equations*. You may recall, rational is synonymous with fractions. We now consider equations where the variable/s may occur in the denominator, such as

$$\frac{1}{15x} + 4 = \frac{1}{6x}$$

One very reliable option is to 'kill' the denominators. This can be accomplished by multiplying both sides by the LCM of the denominators. The rest of the steps will seem familiar as eliminating the denominators will turn these equations into polynomial equations.

$$\begin{aligned}
\frac{1}{15x}+4&=\frac{1}{6x} && \text{(given)}\\
30x\left(\frac{1}{15x}+4\right)&=30x\left(\frac{1}{6x}\right) && \text{(CLM)}\\
30x\cdot\frac{1}{15x}+30x\cdot 4&=30x\cdot\frac{1}{6x} && \text{(DL)}\\
2+120x&=5 && \text{(BI)}\\
120x&=3 && \text{(CLA, BI)}\\
x&=\frac{3}{120}=\frac{1}{40} && \text{(KTC, BI)}
\end{aligned}$$

0.7.1 Exercises

1. Solve the equation:

$$|9y+7|=13$$

2. Solve the equation:

$$|4w+9|=9$$

3. Solve the equation:

$$|6v+19|=9$$

4. Solve the equation:

$$|6z+6|=2$$

5. Solve the equation:

$$|5q|=1$$

6. Solve the equation:

$$\left|\frac{z}{2}+4\right|=3-4z$$

7. Solve the equation:

$$\frac{1}{2v}=\frac{1}{6v^2}+\frac{1}{6v}$$

8. Solve the equation:

$$\frac{7}{r}+1=\frac{1}{r}+5$$

9. Solve the equation:

$$\frac{1}{7x}+\frac{1}{5}=\frac{1}{7x}+\frac{1}{35}$$

10. Solve the equation:

$$\frac{1}{2r}+\frac{1}{4}=\frac{1}{3r}$$

11. Solve the system of equations:

$$\begin{aligned}
5x+3y&=67 && \text{(EQ:A)}\\
5x+2y&=58 && \text{(EQ:B)}
\end{aligned}$$

12. Solve the system of equations:

$$
\begin{aligned}
4x + y &= 6 && \text{(EQ:A)} \\
x + 3y &= 7 && \text{(EQ:B)}
\end{aligned}
$$

13. Solve the system of equations:

$$
\begin{aligned}
-2x - 2y &= -18 && \text{(EQ:A)} \\
-3x - 5y &= -47 && \text{(EQ:B)}
\end{aligned}
$$

14. Solve the system of equations:

$$
\begin{aligned}
-5x - y &= -44 && \text{(EQ:A)} \\
-2x - y &= -17 && \text{(EQ:B)}
\end{aligned}
$$

15. Solve the system of equations:

$$
\begin{aligned}
4x + y &= 35 && \text{(EQ:A)} \\
4x + 4y &= 32 && \text{(EQ:B)}
\end{aligned}
$$

0.8 More On Equations

In this section, we keep the new info to a minimum. Instead, we pause savor some of our triumphs. We look around and find some playful situations that we may turn into equations then solve these using our, now very extensive, solving skills.

Consider the following situation. Suppose on a very nice day, in a very nice city you find yourself having nothing better to do than to toss bright red pebbles from the bridge and onto the water.

Moreover, suppose you have read somewhere that the height, h, of the pebble in projectile motion is related to the time t. Roughly speaking, the more seconds that pass, the closer it gets to the ground. Said differently, the larger t the smaller h becomes. Of course, that is just a very rudimentary description of how the height h the elapsed time t are related. A more precise description may be given by an equation that describes with absolute precision how these are related. The equation may look like this.

$$h = -9.8t^2 + 5t + 200$$

where the height, h, is given in feet, and the time t is given in seconds, as counted from the time the pebble is released. Now we have a good scenario. We could have all sorts of fun with quadratic equations, pebbles, and bridges all at once. We could, for example, ask what what the height at exactly $t = 0 sec$.

Example Suppose

$$h = -9.8t^2 + 5t + 200$$

Find the height, h, when $t = 0$.

Solution:

$$\begin{aligned} h &= -9.8t^2 + 5t + 200 && \text{(given)} \\ h &= -9.8(0)^2 + 5(0) + 200 && \text{(substitute)} \\ h &= 200 && \text{(BI)} \end{aligned}$$

We conclude that the height at $t = 0$ [initial height] was 200 feet from sea-level.

Of course, that was just a little practice. We could as exactly what what the height at precisely $t = 2.35$ seconds.

Example Suppose

$$h = -9.8t^2 + 5t + 200$$

Find the height, h, when $t = 2.35$

Solution:

$$\begin{aligned} h &= -9.8t^2 + 5t + 200 && \text{(given)} \\ h &= -9.8(2.35)^2 + 5(2.35) + 200 && \text{(substitute)} \\ h &= 157.6295 && \text{(BI)} \end{aligned}$$

We conclude that the height at $t = 2.35$ was 157.6295 feet from sea-level.

We could turn the question around a bit. This time we suppose we want to know at exactly what time will the height, h, be precisely 75 feet from sea-level.?

Example Suppose

$$h = -9.8t^2 + 5t + 200$$

Find the time, t, when $h = 75$

Solution:

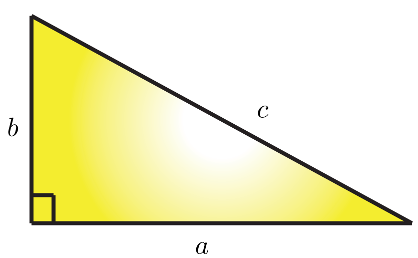
$$\begin{aligned} h &= -9.8t^2 + 5t + 200 && \text{(given)} \\ 75 &= -9.8t^2 + 5t + 200 && \text{(substitute)} \\ 0 &= -9.8t^2 + 5t + 125 && \text{(CLA, Bi)} \\ t &= \frac{-5 \pm \sqrt{5^2 - 4(-9.8)(125)}}{2(-9.8)} && \text{(QF)} \\ t &\approx -3.32543 \quad \text{OR} \quad t \approx 3.83563 && \text{(calculator)} \end{aligned}$$

Assuming we are not going back in time,$t \approx -3.32543$ is not a good choice. The only viable choice here is $t \approx 3.83563\,sec$. We conclude that at $t \approx 3.83563\,sec$ the height, h was precisely 75 feet from sea-level.

Now we switch it up the flavor just a bit. Approximately 2500 years ago, at the height of the glorious ancient Greek civilization, triangles were in vogue. One of the most well known theorems about triangles, made famous at that times is the Pythagorean Theorem. It states that on a right triangle, the square of the longer side [called the *hypotenuse*] is equal to the sum of the squares of the other two sides. We will soon study this famous theorem at length, including its proof. For the moment, we will use it as a way musing ourselves with the things it helps us learn out about the world around us. We first state it.

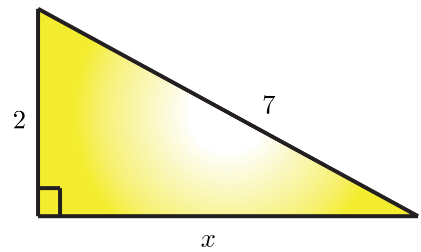
Theorem 0.8.1. *The Pythagorean Theorem says if a triangle has sides of real values, a, b, and c where c is the hypotenuse, then*

$$c^2 = a^2 + b^2$$



One of the consequences of the Pythagorean Theorem is: so long as we know a little about the other two sides, it helps us figure out the third side.

Example Determine the value for the missing side of the triangle:



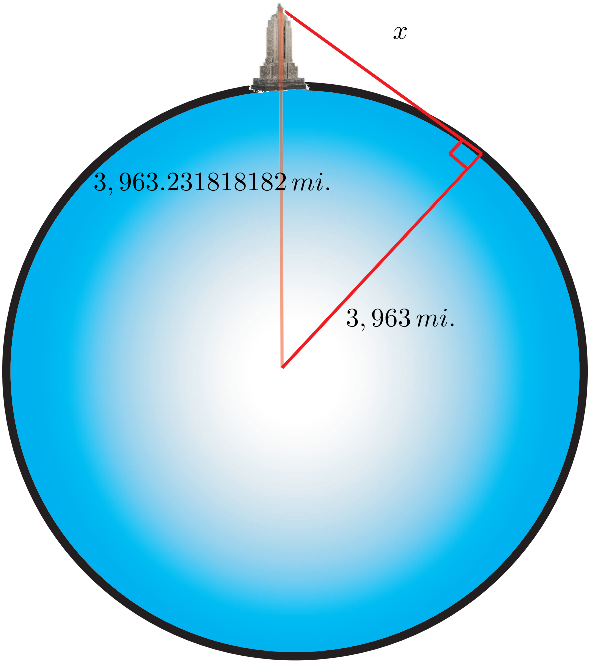
Solution:

$$
\begin{aligned}
7^2 &= 2^2 + x^2 && \text{(Pyth)}\\
49 &= 4 + x^2 && \text{(BI)}\\
x^2 &= 45 && \text{(CLA, BI)}\\
x &= \pm\sqrt{45} && \text{(SRP)}
\end{aligned}
$$

If we can assume the side x is positive then it must be $x = \sqrt{45}$

Here is a slightly more musing observation about the world we live in.. all thanks to the classic Pythagorean Theorem.

Example It is estimated that the earth's radius measures approximately $3,963$ miles. The measurement from the ground to the 102nd floor observatory is 1,224 feet (about 0.231818182 miles). From the observatory, on a clear day how far, how many miles, could one see onto the horizon?



Solution:

$$
\begin{aligned}
(3,963.231818182)^2 &= (3,963)^2 + x^2 && \text{(Pyth)}\\
x^2 &= 1837.44 && \text{(CLA, BI)}\\
x &= \pm\sqrt{1837.44} && \text{(SRP)}
\end{aligned}
$$

If we can assume the side x is positive then it must be that the horizon appear $x = \sqrt{1837.44} \approx 42.8654\,miles$ away from the observatory deck! Yippie Kaee Yaeeehhh!

Now we move on to a different type of observation about our world. This math class is not about money, but if it was, we would allow ourself a few guiltless moments to trifle with such mundane matters. As in....

Example Suppose I have 10 coins, nickels and dimes, for a total of 80 cents. How many of the 10 coins are nickels?

Solution:

We first assign some variables to our situation. Lets call the number of nickels, 'n', and the number of dimes, 'd'. Th first hint provided is that there are a total of 10 coins. This translates into an equation, namely:

$$n + d = 10$$

The second hint is that the total value is 80 cents. This translates to:

$$5n + 10d = 80$$

Now we have ourselves a nice linear 2x2 system, the type we solved in previous sections. Recall we solved by substitution. We solve for n on the first equation to get $n = 10 - d$, then we substitute this into the other equation to get

$$\begin{aligned}
5n + 10d &= 80 && \text{(given EQ:B)}\\
5(10 - d) + 10d &= 80 && \text{(sub)}\\
50 + -5d + 10d &= 80 && \text{(DL, BI)}\\
50 + 5d &= 80 && \text{(BI)}\\
5d &= 30 && \text{(BI)}\\
d &= 30/5 = 6 && \text{(KTC, BI)}
\end{aligned}$$

Thus the number of dimes is $d = 6$ since there are 10 coins that leaves one 4 other coins so the number of nickels must be $n = 4$

0.8.1 Exercises

1. Suppose an object in projectile motion has height, h, described by the equation:

$$h = -15t^2 + 300t + 350$$

where t is the number of seconds from launch, and h is measured in feet. Find the heights h at times $t = 10$, $t = 13$ and $t = 19$

2. Suppose an object in projectile motion has height, h, described by the equation:

$$h = -10t^2 + 120t + 120$$

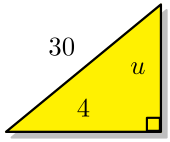
where t is the number of seconds from launch, and h is measured in feet. Find the heights h at times $t = 1$, $t = 5$ and $t = 8$

3. Suppose an object in projectile motion has height, h, described by the equation:

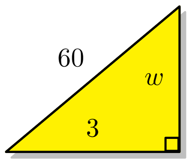
$$h = -14t^2 + 40t + 300$$

where t is the number of seconds from launch, and h is measured in feet. Find the time t when the object hits the ground.

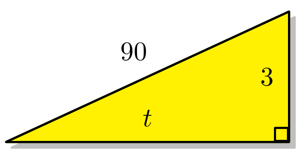
4. Determine the missing side of the triangle [not drawn to scale].



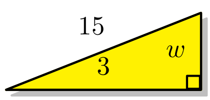
5. Determine the missing side of the triangle [not drawn to scale].



6. Determine the missing side of the triangle [not drawn to scale].



7. Determine the missing side of the triangle [not drawn to scale].



8. Assuming the earth is perfectly round with radius 3963 miles, with clear weather etc. How far can one see onto the horizon from a building that is 1050 feet above ground level?

9. Assuming the earth is perfectly round with radius 3963 miles, with clear weather etc. How far can one see onto the horizon from a building that is 480 feet above ground level?

10. Assuming the earth is perfectly round with radius 3963 miles, with clear weather etc. How far can one see onto the horizon from a building that is 520 feet above ground level?

11. Assuming the earth is perfectly round with radius 3963 miles, with clear weather etc. How far can one see onto the horizon from a building that is 1050 feet above ground level?

12. Suppose 28 coins consiting of pennies and quarters amount to \$4.84. How many of each type of coins are there?

13. Suppose 32 coins consiting of quarters and nickels amount to \$5.00. How many of each type of coins are there?

14. Suppose 28 coins consiting of half-dollars and nickels amount to \$7.70. How many of each type of coins are there?

15. Suppose 27 coins consiting of nickels and pennies amount to \$0.830. How many of each type of coins are there?

0.9 Chapter 5 Summary

Solving Linear Equations

name	from	to
Free the terms	$3(x+1) = \frac{x+1}{2}$	$3x+1 = \frac{1}{2}x + \frac{1}{2}$
Shuffle the terms	$3x+1 = \frac{1}{2}x + \frac{1}{2}$	$3x + -\frac{1}{2}x = -1 + \frac{1}{2}$
Collect the terms	$3x + -\frac{1}{2}x = -1 + \frac{1}{2}$	$\left(3 + -\frac{1}{2}\right)x = -1 + \frac{1}{2}$
Kill the Coefficient	$\left(3 + -\frac{1}{2}\right)x = -1 + \frac{1}{2}$	$x = \frac{-1+\frac{1}{2}}{3+-\frac{1}{2}}$

Theorems

name	symbol	says
Zero Factor Theorem	*ZFT*	$A \cdot B = 0 \implies A = 0 \text{ or } B = 0$
Square Root Property	*SRP*	$x^2 = a \implies x = \pm\sqrt{a}$
Absolute Value Theorem	*AVT*	$\lvert x\rvert = a \implies x = a \text{ or } x = -a$
Quadratic Formula	*QF*	$ax^2 + bx + c = 0 \implies x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

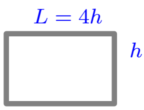
Strategies

example	strategy	such as
$x^2 + 3x + 2 = 0$	*Factors & ZFT*	$(x+2)(x+1) = 0$
$x^2 + 3x + 2 = 0$	*Complete PP2 & SRP*	$\left(x + \frac{3}{2}\right)^2 = \frac{9}{4}$
$x^2 + 3x + 2 = 0$	*Quadratic Formula*	$x = \frac{-3\pm\sqrt{3^2-4\cdot1\cdot2}}{2\cdot1}$
$x^2 - 9 = 0$	*Recognize Famous*	$(x-3)(x+3) = 0$

Some Answers

Section 0.1

1.



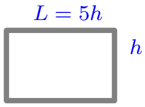
$$\begin{aligned}
\text{total fence is equal to 250 ft.} & & \text{(given)}\\
h + h + 4h + 4h &= 250ft & \text{(interpret)}\\
1 \cdot h + 1 \cdot h + 4h + 4h &= 250ft & \text{(miD)}\\
(1 + 1 + 2 + 2)h &= 250ft & \text{(DL)}\\
h &= \frac{250ft}{1 + 1 + 4 + 4} & \text{(KTC)}\\
h &= \frac{250ft}{10} & \text{(KTC)}\\
h &= 25ft & \text{(BI)}
\end{aligned}$$

Since the length is 4 times as much it must be $100ft$ long.

2.

$$\begin{aligned}
9c &= 5(f - 32) & \text{(given)}\\
9 \cdot 28 &= 5(f - 32) & \text{(sub c=28)}\\
9 \cdot 28 &= 5f - 5 \cdot 32 & \text{(Bi, DL)}\\
5f - 5 \cdot 32 &= 9 \cdot 28 & \text{(SP)}\\
5f - 5 \cdot 32 + 5 \cdot 32 &= 9 \cdot 28 + 5 \cdot 32 & \text{(CLA)}\\
5f &= 9 \cdot 28 + cb5 \cdot 32 & \text{(BI)}\\
5f &= 412 & \text{(BI)}\\
f &= \frac{412}{5} & \text{(KTC)}\\
f &= 82.4 & \text{(calc.)}
\end{aligned}$$

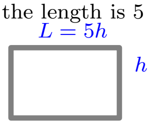
3.



$$\begin{aligned}
\text{total fence is equal } & \text{ to 420 ft.} && \text{(given)}\\
h + h + 5h + 5h &= 420ft && \text{(interpret)}\\
1 \cdot h + 1 \cdot h + 5h + 5h &= 420ft && \text{(miD)}\\
(1 + 1 + 2 + 2)h &= 420ft && \text{(DL)}\\
h &= \frac{420ft}{1 + 1 + 5 + 5} && \text{(KTC)}\\
h &= \frac{420ft}{12} && \text{(KTC)}\\
h &= 35ft && \text{(BI)}
\end{aligned}$$

Since the length is 5 times as much it must be $175ft$ long.

4.



$$\begin{aligned}
\text{total fence is equal } & \text{ to 360 ft.} && \text{(given)}\\
h + h + 5h + 5h &= 360ft && \text{(interpret)}\\
1 \cdot h + 1 \cdot h + 5h + 5h &= 360ft && \text{(miD)}\\
(1 + 1 + 2 + 2)h &= 360ft && \text{(DL)}\\
h &= \frac{360ft}{1 + 1 + 5 + 5} && \text{(KTC)}\\
h &= \frac{360ft}{12} && \text{(KTC)}\\
h &= 30ft && \text{(BI)}
\end{aligned}$$

Since the length is 5 times as much it must be $150ft$ long.

5.

$$\begin{aligned}
9c &= 5(f - 32) && \text{(given)}\\
9 \cdot 6 &= 5(f - 32) && \text{(sub c=6)}\\
9 \cdot 6 &= 5f - 5 \cdot 32 && \text{(Bi, DL)}\\
5f - 5 \cdot 32 &= 9 \cdot 6 && \text{(SP)}\\
5f - 5 \cdot 32 + {\color{blue}5 \cdot 32} &= 9 \cdot 6 + {\color{blue}5 \cdot 32} && \text{(CLA)}\\
5f &= 9 \cdot 6 + cb5 \cdot 32 && \text{(BI)}\\
5f &= 214 && \text{(BI)}\\
f &= \frac{214}{5} && \text{(KTC)}\\
f &= 42.8 && \text{(calc.)}
\end{aligned}$$

6.

$$\begin{aligned}
9c &= 5(f-32) && \text{(given)}\\
9\cdot 42 &= 5(f-32) && \text{(sub c=42)}\\
9\cdot 42 &= 5f-5\cdot 32 && \text{(Bi, DL)}\\
5f-5\cdot 32 &= 9\cdot 42 && \text{(SP)}\\
5f-5\cdot 32+5\cdot 32 &= 9\cdot 42+5\cdot 32 && \text{(CLA)}\\
5f &= 9\cdot 42+cb5\cdot 32 && \text{(BI)}\\
5f &= 538 && \text{(BI)}\\
f &= \frac{538}{5} && \text{(KTC)}\\
f &= 108. && \text{(calc.)}
\end{aligned}$$

7.

$$\begin{aligned}
9c &= 5(f-32) && \text{(given)}\\
9\cdot 17 &= 5(f-32) && \text{(sub c=17)}\\
9\cdot 17 &= 5f-5\cdot 32 && \text{(Bi, DL)}\\
5f-5\cdot 32 &= 9\cdot 17 && \text{(SP)}\\
5f-5\cdot 32+5\cdot 32 &= 9\cdot 17+5\cdot 32 && \text{(CLA)}\\
5f &= 9\cdot 17+cb5\cdot 32 && \text{(BI)}\\
5f &= 313 && \text{(BI)}\\
f &= \frac{313}{5} && \text{(KTC)}\\
f &= 62.6 && \text{(calc.)}
\end{aligned}$$

8.

$$\begin{aligned}
9c &= 5(f-32) && \text{(given)}\\
9\cdot 38 &= 5(f-32) && \text{(sub c=38)}\\
9\cdot 38 &= 5f-5\cdot 32 && \text{(Bi, DL)}\\
5f-5\cdot 32 &= 9\cdot 38 && \text{(SP)}\\
5f-5\cdot 32+5\cdot 32 &= 9\cdot 38+5\cdot 32 && \text{(CLA)}\\
5f &= 9\cdot 38+cb5\cdot 32 && \text{(BI)}\\
5f &= 502 && \text{(BI)}\\
f &= \frac{502}{5} && \text{(KTC)}\\
f &= 100. && \text{(calc.)}
\end{aligned}$$

9.

$$\begin{aligned}
\text{3 times the sum of } V \text{ and 7 is equalto 6} && \text{(given)}\\
3(V+7) &= 6 && \text{(interpret)}\\
3V+21 &= 6 && \text{(DL, BI)}\\
3V+21+-21 &= 6+-21 && \text{(CLA)}\\
3V &= -15 && \text{(BI)}\\
V &= \frac{-15}{3} && \text{(KTC)}\\
V &= -5 && \text{(BI)}
\end{aligned}$$

10.

$$\begin{aligned}
\text{3 times the sum of } \star \text{ and 6 is equalto } -9 && \text{(given)}\\
3(\star+6) &= -9 && \text{(interpret)}\\
3\star+18 &= -9 && \text{(DL, BI)}\\
3\star+18+-18 &= -9+-18 && \text{(CLA)}\\
3\star &= -27 && \text{(BI)}\\
\star &= \frac{-27}{3} && \text{(KTC)}\\
\star &= -9 && \text{(BI)}
\end{aligned}$$

Section 0.2

1.

$$\begin{aligned}
28+(-6\cdot q) &= q(-9+q) && \text{(given)}\\
28-6q &= q^2-9q && \text{(BI)}\\
-q^2+3q+28 &= 0 && \text{(BI)}\\
(q-7)\,(-q-4) &= 0 && \text{(BI)}\\
q-7=0 \qquad \text{OR} \qquad -q-4 &= 0 && \text{(ZFT, BI)}\\
q=7 \qquad \text{OR} \qquad q &= -4 && \text{(BI)}
\end{aligned}$$

2.

$$\begin{aligned}
6w^2 - 2w - 9 &= 7w - 3 && \text{(given)}\\
6w^2 + -9w + -6 &= 0 && \text{(CLA, BI)}\\
6w^2 + 3w + -12w + -6 &= 0 && \text{(BI, the Split)}\\
(3w)(2w+1) + (-6)(2w+1) &= 0 && \text{(DL, BI)}\\
(3w + -6)(2w+1) &= 0 && \text{(DI)}\\
3w + -6 = 0 \qquad \text{OR} \qquad 2w + 1 &= 0 && \text{(cbZFT)}\\
w = 2 \qquad \text{OR} \qquad w &= -\frac{1}{2} && \text{(DI)}
\end{aligned}$$

3.

$$\begin{aligned}
base \cdot height &= area && \text{(assume)}\\
(8y - 49)\,(y - 2) &= 35 && \text{(given, [substitute])}\\
8y^2 - 65y + 98 &= 35 && \text{(given, [FOIL, BI])}\\
8y^2 - 65y + 63 &= 0 && \text{(BI)}\\
(y - 7)\,(8y - 9) &= 0 && \text{(BI)}\\
y - 7 = 0 \qquad \text{OR} \qquad 8y - 9 &= 0 && \text{(ZFT, BI)}\\
y = 7 \qquad \text{OR} \qquad y &= \frac{9}{8} && \text{(BI)}
\end{aligned}$$

Thus the dimensions for the rectable are $base \times height = (8y - 49) \times (y - 2)$ are

$$7 \times 5 \qquad \text{OR} \qquad -40 \times -\frac{7}{8}$$

We will assume the dimensions are the positive dimensions $7ft. \times 5ft.$.

4.

$$\begin{aligned}
t^2 + 41t + 29 &= 29t - 6 && \text{(given)}\\
t^2 + 12t + 35 &= 0 && \text{(CLA, BI)}\\
t^2 + 7t + 5t + 35 &= 0 && \text{(BI, the Split)}\\
(t)(t+7) + (5)(t+7) &= 0 && \text{(DL, BI)}\\
(t+5)(t+7) &= 0 && \text{(DI)}\\
t + 5 = 0 \qquad \text{OR} \qquad t + 7 &= 0 && \text{(cbZFT)}\\
t = -5 \qquad \text{OR} \qquad t &= -7 && \text{(DI)}
\end{aligned}$$

5.

$$\begin{aligned} base \cdot height &= area && \text{(assume)} \\ (4w-19)(w-3) &= 15 && \text{(given, [substitute])} \\ 4w^2 - 31w + 57 &= 15 && \text{(given, [FOIL, BI])} \\ 4w^2 - 31w + 42 &= 0 && \text{(BI)} \\ (w-6)(4w-7) &= 0 && \text{(BI)} \\ w-6=0 \qquad \text{OR} \qquad 4w-7 &= 0 && \text{(ZFT, BI)} \\ w=6 \qquad \text{OR} \qquad w &= \frac{7}{4} && \text{(BI)} \end{aligned}$$

Thus the dimensions for the rectable are $base \times height = (4w-19) \times (w-3)$ are

$$5 \times 3 \qquad \text{OR} \qquad -12 \times -\frac{5}{4}$$

We will assume the dimensions are the positive dimensions $5ft. \times 3ft.$.

6.

$$\begin{aligned} -14 + (8 \cdot u) &= u(-1+u) && \text{(given)} \\ 8u - 14 &= u^2 - u && \text{(BI)} \\ -u^2 + 9u - 14 &= 0 && \text{(BI)} \\ (u-7)(2-u) &= 0 && \text{(BI)} \\ u-7=0 \qquad \text{OR} \qquad 2-u &= 0 && \text{(ZFT, BI)} \\ u=7 \qquad \text{OR} \qquad u &= 2 && \text{(BI)} \end{aligned}$$

7.

$$\begin{aligned} 6r^2 + 22r + 12 &= 0 && \text{(given)} \\ 6r^2 + 4r + 18r + 12 &= 0 && \text{(BI, the Split)} \\ (2r)(3r+2) + (6)(3r+2) &= 0 && \text{(DL, BI)} \\ (2r+6)(3r+2) &= 0 && \text{(DI)} \\ 2r+6=0 \qquad \text{OR} \qquad 3r+2 &= 0 && \text{(cbZFT)} \\ r=-3 \qquad \text{OR} \qquad r &= -\frac{2}{3} && \text{(DI)} \end{aligned}$$

8.

$$\begin{aligned} 48 + (-3 \cdot v) &= v(-1+v) && \text{(given)} \\ 48 - 3v &= v^2 - v && \text{(BI)} \\ -v^2 - 2v + 48 &= 0 && \text{(BI)} \\ (v-6)(-v-8) &= 0 && \text{(BI)} \\ v-6=0 \qquad \text{OR} \qquad -v-8 &= 0 && \text{(ZFT, BI)} \\ v=6 \qquad \text{OR} \qquad v &= -8 && \text{(BI)} \end{aligned}$$

9.

$$
\begin{aligned}
6x^2 + x + -15 &= 0 && \text{(given)}\\
6x^2 + 10x + -9x + -15 &= 0 && \text{(BI, the Split)}\\
(2x)(3x+5) + (-3)(3x+5) &= 0 && \text{(DL, BI)}\\
(2x + -3)(3x+5) &= 0 && \text{(DI)}\\
2x + -3 = 0 \qquad \text{OR} \qquad 3x + 5 &= 0 && \text{(cbZFT)}\\
x = \frac{3}{2} \qquad \text{OR} \qquad x = -\frac{5}{3} & && \text{(DI)}
\end{aligned}
$$

10.

$$
\begin{aligned}
3q^2 + 0 + -12 &= 0 && \text{(given)}\\
3q^2 + 6q + -6q + -12 &= 0 && \text{(BI, the Split)}\\
(3q)(q+2) + (-6)(q+2) &= 0 && \text{(DL, BI)}\\
(3q + -6)(q+2) &= 0 && \text{(DI)}\\
3q + -6 = 0 \qquad \text{OR} \qquad q + 2 &= 0 && \text{(cbZFT)}\\
q = 2 \qquad \text{OR} \qquad q = -2 & && \text{(DI)}
\end{aligned}
$$

11.

$$
\begin{aligned}
-27 + (-8 \cdot r) &= r(-20 + r) && \text{(given)}\\
-8r - 27 &= r^2 - 20r && \text{(BI)}\\
-r^2 + 12r - 27 &= 0 && \text{(BI)}\\
(r-9)\,(3-r) &= 0 && \text{(BI)}\\
r - 9 = 0 \qquad \text{OR} \qquad 3 - r &= 0 && \text{(ZFT, BI)}\\
r = 9 \qquad \text{OR} \qquad r &= 3 && \text{(BI)}
\end{aligned}
$$

12.

$$
\begin{aligned}
z^2 + 4z + -21 &= 0 && \text{(given)}\\
z^2 + 7z + -3z + -21 &= 0 && \text{(BI, the Split)}\\
(z)(z+7) + (-3)(z+7) &= 0 && \text{(DL, BI)}\\
(z + -3)(z+7) &= 0 && \text{(DI)}\\
z + -3 = 0 \qquad \text{OR} \qquad z + 7 &= 0 && \text{(cbZFT)}\\
z = 3 \qquad \text{OR} \qquad z = -7 & && \text{(DI)}
\end{aligned}
$$

13.

$$\begin{aligned}
2q^2 + 16q - 23 &= 19q - 3 && \text{(given)}\\
2q^2 + -3q + -20 &= 0 && \text{(CLA, BI)}\\
2q^2 + 5q + -8q + -20 &= 0 && \text{(BI, the Split)}\\
(q)(2q+5) + (-4)(2q+5) &= 0 && \text{(DL, BI)}\\
(q + -4)(2q+5) &= 0 && \text{(DI)}\\
q + -4 = 0 \qquad \text{OR} \qquad 2q + 5 &= 0 && \text{(cbZFT)}\\
q = 4 \qquad \text{OR} \qquad q &= -\frac{5}{2} && \text{(DI)}
\end{aligned}$$

14.

$$\begin{aligned}
9u^2 - 10u + 4 &= 11u - 6 && \text{(given)}\\
9u^2 + -21u + 10 &= 0 && \text{(CLA, BI)}\\
9u^2 + -15u + -6u + 10 &= 0 && \text{(BI, the Split)}\\
(3u)(3u + -5) + (-2)(3u + -5) &= 0 && \text{(DL, BI)}\\
(3u + -2)(3u + -5) &= 0 && \text{(DI)}\\
3u + -2 = 0 \qquad \text{OR} \qquad 3u + -5 &= 0 && \text{(cbZFT)}\\
u = \frac{2}{3} \qquad \text{OR} \qquad u &= \frac{5}{3} && \text{(DI)}
\end{aligned}$$

15.

$$\begin{aligned}
6v^2 + 17v - 1 &= 7v + 3 && \text{(given)}\\
6v^2 + 10v + -4 &= 0 && \text{(CLA, BI)}\\
6v^2 + -2v + 12v + -4 &= 0 && \text{(BI, the Split)}\\
(2v)(3v + -1) + (4)(3v + -1) &= 0 && \text{(DL, BI)}\\
(2v + 4)(3v + -1) &= 0 && \text{(DI)}\\
2v + 4 = 0 \qquad \text{OR} \qquad 3v + -1 &= 0 && \text{(cbZFT)}\\
v = -2 \qquad \text{OR} \qquad v &= \frac{1}{3} && \text{(DI)}
\end{aligned}$$

16.

$$\begin{aligned}
3q^2 + 35q + 7 &= 23q - 5 && \text{(given)}\\
3q^2 + 12q + 12 &= 0 && \text{(CLA, BI)}\\
3q^2 + 6q + 6q + 12 &= 0 && \text{(BI, the Split)}\\
(3q)(q+2) + (6)(q+2) &= 0 && \text{(DL, BI)}\\
(3q + 6)(q+2) &= 0 && \text{(DI)}\\
3q + 6 = 0 \qquad \text{OR} \qquad q + 2 &= 0 && \text{(cbZFT)}\\
q = -2 \qquad \text{OR} \qquad q &= -2 && \text{(DI)}
\end{aligned}$$

17.

$$\begin{aligned}
4v^2 + 9v - 13 &= 5v - 5 && \text{(given)}\\
4v^2 + 4v + -8 &= 0 && \text{(CLA, BI)}\\
4v^2 + -4v + 8v + -8 &= 0 && \text{(BI, the Split)}\\
(2v)(2v + -2) + (4)(2v + -2) &= 0 && \text{(DL, BI)}\\
(2v + 4)(2v + -2) &= 0 && \text{(DI)}\\
2v + 4 = 0 \qquad \text{OR} \qquad 2v + -2 &= 0 && \text{(cbZFT)}\\
v = -2 \qquad \text{OR} \qquad v &= 1 && \text{(DI)}
\end{aligned}$$

18.

$$\begin{aligned}
9 + (-2 \cdot r) &= r(-10 + r) && \text{(given)}\\
9 - 2r &= r^2 - 10r && \text{(BI)}\\
-r^2 + 8r + 9 &= 0 && \text{(BI)}\\
(r - 9)\,(-r - 1) &= 0 && \text{(BI)}\\
r - 9 = 0 \qquad \text{OR} \qquad -r - 1 &= 0 && \text{(ZFT, BI)}\\
r = 9 \qquad \text{OR} \qquad r &= -1 && \text{(BI)}
\end{aligned}$$

19.

$$\begin{aligned}
base \cdot height &= area && \text{(assume)}\\
(6u - 17)\,(u - 2) &= 14 && \text{(given, [substitute])}\\
6u^2 - 29u + 34 &= 14 && \text{(given, [FOIL, BI])}\\
6u^2 - 29u + 20 &= 0 && \text{(BI)}\\
(u - 4)\,(6u - 5) &= 0 && \text{(BI)}\\
u - 4 = 0 \qquad \text{OR} \qquad 6u - 5 &= 0 && \text{(ZFT, BI)}\\
u = 4 \qquad \text{OR} \qquad u &= \frac{5}{6} && \text{(BI)}
\end{aligned}$$

Thus the dimensions for the rectable are $base \times height = (6u - 17) \times (u - 2)$ are

$$7 \times 2 \qquad \text{OR} \qquad -12 \times -\frac{7}{6}$$

We will assume the dimensions are the positive dimensions $7ft. \times 2ft.$.

20.

$$\begin{aligned}
4y^2 + 2y + -6 &= 0 && \text{(given)}\\
4y^2 + 6y + -4y + -6 &= 0 && \text{(BI, the Split)}\\
(2y)(2y + 3) + (-2)(2y + 3) &= 0 && \text{(DL, BI)}\\
(2y + -2)(2y + 3) &= 0 && \text{(DI)}\\
2y + -2 = 0 \qquad \text{OR} \qquad 2y + 3 &= 0 && \text{(cbZFT)}\\
y = 1 \qquad \text{OR} \qquad y &= -\frac{3}{2} && \text{(DI)}
\end{aligned}$$

21.

$$\begin{aligned}
9z^2 - 12z - 11 &= 3z + 3 && \text{(given)}\\
9z^2 + -15z + -14 &= 0 && \text{(CLA, BI)}\\
9z^2 + -21z + 6z + -14 &= 0 && \text{(BI, the Split)}\\
(3z)(3z + -7) + (2)(3z + -7) &= 0 && \text{(DL, BI)}\\
(3z + 2)(3z + -7) &= 0 && \text{(DI)}\\
3z + 2 = 0 \qquad \text{OR} \qquad 3z + -7 &= 0 && \text{(cbZFT)}\\
z = -\frac{2}{3} \qquad \text{OR} \qquad z &= \frac{7}{3} && \text{(DI)}
\end{aligned}$$

22.

$$\begin{aligned}
3t^2 + 22t + 7 &= 0 && \text{(given)}\\
3t^2 + t + 21t + 7 &= 0 && \text{(BI, the Split)}\\
(t)(3t + 1) + (7)(3t + 1) &= 0 && \text{(DL, BI)}\\
(t + 7)(3t + 1) &= 0 && \text{(DI)}\\
t + 7 = 0 \qquad \text{OR} \qquad 3t + 1 &= 0 && \text{(cbZFT)}\\
t = -7 \qquad \text{OR} \qquad t &= -\frac{1}{3} && \text{(DI)}
\end{aligned}$$

23.

$$\begin{aligned}
x^2 + -2x + -35 &= 0 && \text{(given)}\\
x^2 + -7x + 5x + -35 &= 0 && \text{(BI, the Split)}\\
(x)(x + -7) + (5)(x + -7) &= 0 && \text{(DL, BI)}\\
(x + 5)(x + -7) &= 0 && \text{(DI)}\\
x + 5 = 0 \qquad \text{OR} \qquad x + -7 &= 0 && \text{(cbZFT)}\\
x = -5 \qquad \text{OR} \qquad x &= 7 && \text{(DI)}
\end{aligned}$$

24.

$$\begin{aligned}
6t^2 - 4t - 47 &= 5t - 5 && \text{(given)}\\
6t^2 + -9t + -42 &= 0 && \text{(CLA, BI)}\\
6t^2 + 12t + -21t + -42 &= 0 && \text{(BI, the Split)}\\
(2t)(3t + 6) + (-7)(3t + 6) &= 0 && \text{(DL, BI)}\\
(2t + -7)(3t + 6) &= 0 && \text{(DI)}\\
2t + -7 = 0 \qquad \text{OR} \qquad 3t + 6 &= 0 && \text{(cbZFT)}\\
t = \frac{7}{2} \qquad \text{OR} \qquad t &= -2 && \text{(DI)}
\end{aligned}$$

25.

$$
\begin{aligned}
3q^2 + -5q + -28 &= 0 && \text{(given)}\\
3q^2 + 7q + -12q + -28 &= 0 && \text{(BI, the Split)}\\
(q)(3q+7) + (-4)(3q+7) &= 0 && \text{(DL, BI)}\\
(q+-4)(3q+7) &= 0 && \text{(DI)}\\
q + -4 = 0 \qquad \text{OR} \qquad 3q+7 &= 0 && \text{(cbZFT)}\\
q = 4 \qquad \text{OR} \qquad q &= -\frac{7}{3} && \text{(DI)}
\end{aligned}
$$

Section 0.3

1. $\sqrt[3]{-42} \approx -3.48$

2. $\sqrt{\frac{4}{3}} \approx 1.15$

3.

$$
\begin{aligned}
(12t+4)^2 &= \frac{16}{9} && \text{(given)}\\
12t + 4 &= \pm\sqrt{\frac{16}{9}} && \text{(by SRP)}\\
12t + 4 &= \pm\frac{4}{3} && \text{(def of } \sqrt{\ }\text{)}\\
12t &= -4 \pm \frac{4}{3} && \text{(CLA, BI)}\\
t &= \frac{-4 \pm \frac{4}{3}}{12} && \text{(KTC)}
\end{aligned}
$$

4. $\sqrt[3]{-\frac{64}{125}} = -\frac{4}{5}$

5.

$$
\begin{aligned}
(t+14)^2 &= \frac{9}{16} && \text{(given)}\\
t + 14 &= \pm\sqrt{\frac{9}{16}} && \text{(by SRP)}\\
t + 14 &= \pm\frac{3}{4} && \text{(def of } \sqrt{\ }\text{)}\\
t &= -14 \pm \frac{3}{4} && \text{(CLA, BI)}
\end{aligned}
$$

6.

$$\begin{aligned} (y+13)^2 &= 9 && \text{(given)} \\ y+13 &= \pm\sqrt{9} && \text{(by SRP)} \\ y+13 &= \pm 3 && \text{(def of } \sqrt{\ }\text{)} \\ y &= -13 \pm 3 && \text{(CLA, BI)} \end{aligned}$$

7. $\sqrt{49} = 7$

8. $\sqrt[3]{-1} = -1$

9.

$$\begin{aligned} (12v+2)^2 &= 1 && \text{(given)} \\ 12v+2 &= \pm\sqrt{1} && \text{(by SRP)} \\ 12v+2 &= \pm 1 && \text{(def of } \sqrt{\ }\text{)} \\ 12v &= -2 \pm 1 && \text{(CLA, BI)} \\ v &= \frac{-2 \pm 1}{12} && \text{(KTC)} \end{aligned}$$

10.

$$\begin{aligned} 9(v)^2 &= \frac{25}{4} && \text{(given)} \\ (v)^2 &= \frac{\frac{25}{4}}{9} && \text{(KTC)} \\ v &= \pm\sqrt{\frac{\frac{25}{4}}{9}} && \text{(by SRP)} \end{aligned}$$

11.

$$\begin{aligned} W^2 &= 64 && \text{(given)} \\ W &= \pm\sqrt{64} && \text{(by SRP)} \\ W &= \pm 8 && \text{(def of } \sqrt{\ }\text{)} \end{aligned}$$

12. $\sqrt{\frac{4}{5}} \approx 0.894$

13.

$$\begin{aligned} (v+8)^2 &= 16 && \text{(given)} \\ v+8 &= \pm\sqrt{16} && \text{(by SRP)} \\ v+8 &= \pm 4 && \text{(def of } \sqrt{\ }\text{)} \\ v &= -8 \pm 4 && \text{(CLA, BI)} \end{aligned}$$

14. $\sqrt{35} \approx 5.92$

15. $\sqrt[3]{-44} \approx -3.53$

16.

$$\begin{aligned}
(z+7)^2 &= 169 && \text{(given)}\\
z+7 &= \pm\sqrt{169} && \text{(by SRP)}\\
z+7 &= \pm 13 && \text{(def of } \sqrt{\ }\text{)}\\
z &= -7 \pm 13 && \text{(CLA, BI)}
\end{aligned}$$

17.

$$\begin{aligned}
(3u+12)^2 &= \frac{4}{25} && \text{(given)}\\
3u+12 &= \pm\sqrt{\frac{4}{25}} && \text{(by SRP)}\\
3u+12 &= \pm\frac{2}{5} && \text{(def of } \sqrt{\ }\text{)}\\
3u &= -12 \pm \frac{2}{5} && \text{(CLA, BI)}\\
u &= \frac{-12 \pm \frac{2}{5}}{3} && \text{(KTC)}
\end{aligned}$$

18. $\sqrt[3]{\frac{8}{125}} = \frac{2}{5}$

19. $\sqrt{64} = 8$

20.

$$\begin{aligned}
(2x+7)^2 &= 4 && \text{(given)}\\
2x+7 &= \pm\sqrt{4} && \text{(by SRP)}\\
2x+7 &= \pm 2 && \text{(def of } \sqrt{\ }\text{)}\\
2x &= -7 \pm 2 && \text{(CLA, BI)}\\
x &= \frac{-7 \pm 2}{2} && \text{(KTC)}
\end{aligned}$$

Section 0.4

1.

$$\begin{aligned} q^2 &= -\frac{4}{3} && \text{(given)} \\ q &= \pm\sqrt{-\frac{4}{3}} && \text{(by SRP)} \\ q &= \pm\frac{2i}{\sqrt{3}} && \text{(nRad, BI)} \end{aligned}$$

2.

$$\begin{aligned} (i-4)(3i-6) &= 3i^2 - 18i + 24 && \text{(FOIL, BI)} \\ &= 3(-1) - 18\text{i} + 24 && \text{(def } i\text{)} \\ &= 21 - 18i && \text{(BI)} \end{aligned}$$

3. $\sqrt{-15} = i\sqrt{15}$

4. $\sqrt{-3} = i\sqrt{3}$

5.

$$\begin{aligned} (i-7)(3i-5) &= 3i^2 - 26i + 35 && \text{(FOIL, BI)} \\ &= 3(-1) - 26\text{i} + 35 && \text{(def } i\text{)} \\ &= 32 - 26i && \text{(BI)} \end{aligned}$$

6.

$$\begin{aligned} q^2 &= -\frac{2}{5} && \text{(given)} \\ q &= \pm\sqrt{-\frac{2}{5}} && \text{(by SRP)} \\ q &= \pm i\sqrt{\frac{2}{5}} && \text{(nRad, BI)} \end{aligned}$$

7.

$$\begin{aligned} (t-14)^2 &= -14 && \text{(given)} \\ t - 14 &= \pm\sqrt{-14} && \text{(by SRP)} \\ t - 14 &= \pm i\sqrt{14} && \text{(nRad, BI)} \\ t &= 14 \pm i\sqrt{14} && \text{(CLA, BI)} \end{aligned}$$

8.

$$\begin{aligned} (q-8)^2 &= -8 && \text{(given)} \\ q-8 &= \pm\sqrt{-8} && \text{(by SRP)} \\ q-8 &= \pm 2i\sqrt{2} && \text{(nRad, BI)} \\ q &= 8 \pm 2i\sqrt{2} && \text{(CLA, BI)} \end{aligned}$$

9.

$$\begin{aligned} v^2 &= -\frac{4}{5} && \text{(given)} \\ v &= \pm\sqrt{-\frac{4}{5}} && \text{(by SRP)} \\ v &= \pm\frac{2i}{\sqrt{5}} && \text{(nRad, BI)} \end{aligned}$$

10.

$$\begin{aligned} (z-15)^2 &= -2 && \text{(given)} \\ z-15 &= \pm\sqrt{-2} && \text{(by SRP)} \\ z-15 &= \pm i\sqrt{2} && \text{(nRad, BI)} \\ z &= 15 \pm i\sqrt{2} && \text{(CLA, BI)} \end{aligned}$$

11. $\sqrt{-2} = i\sqrt{2}$

12.

$$\begin{aligned} v^2 &= -1 && \text{(given)} \\ v &= \pm\sqrt{-1} && \text{(by SRP)} \\ v &= \pm i && \text{(nRad, BI)} \end{aligned}$$

13.

$$\begin{aligned} (y-15)^2 &= -\frac{3}{2} && \text{(given)} \\ y-15 &= \pm\sqrt{-\frac{3}{2}} && \text{(by SRP)} \\ y-15 &= \pm i\sqrt{\frac{3}{2}} && \text{(nRad, BI)} \\ y &= 15 \pm i\sqrt{\frac{3}{2}} && \text{(CLA, BI)} \end{aligned}$$

14. $\sqrt{-7} = i\sqrt{7}$

15.

$$\begin{aligned} w^2 &= -\frac{5}{3} && \text{(given)} \\ w &= \pm\sqrt{-\frac{5}{3}} && \text{(by SRP)} \\ w &= \pm i\sqrt{\frac{5}{3}} && \text{(nRad, BI)} \end{aligned}$$

16. $\sqrt{-15} = i\sqrt{15}$

17.

$$\begin{aligned} w^2 &= -\frac{4}{3} && \text{(given)} \\ w &= \pm\sqrt{-\frac{4}{3}} && \text{(by SRP)} \\ w &= \pm\frac{2i}{\sqrt{3}} && \text{(nRad, BI)} \end{aligned}$$

18.

$$\begin{aligned} (z-14)^2 &= -1 && \text{(given)} \\ z-14 &= \pm\sqrt{-1} && \text{(by SRP)} \\ z-14 &= \pm i && \text{(nRad, BI)} \\ z &= 14 \pm i && \text{(CLA, BI)} \end{aligned}$$

19.

$$\begin{aligned} (r-7)^2 &= -5 && \text{(given)} \\ r-7 &= \pm\sqrt{-5} && \text{(by SRP)} \\ r-7 &= \pm i\sqrt{5} && \text{(nRad, BI)} \\ r &= 7 \pm i\sqrt{5} && \text{(CLA, BI)} \end{aligned}$$

20. $\sqrt{-\frac{4}{5}} = \frac{2i}{\sqrt{5}}$

Section 0.5

1.

$$
\begin{aligned}
q^2 + q &= -2 && \text{(given)}\\
q^2 + 2 \cdot \frac{1}{2} q &= -2 && \text{(BI)}\\
q^2 + 2 \cdot \frac{1}{2} q + \left(\frac{1}{2}\right)^2 &= -2 + \left(\frac{1}{2}\right)^2 && \text{(CLA)}\\
\left(q + \frac{1}{2}\right)^2 &= -2 + \left(\frac{1}{2}\right)^2 && \text{(PP2)}\\
\left(q + \frac{1}{2}\right)^2 &= -\frac{7}{4} && \text{(Bi)}\\
q + \frac{1}{2} &= \pm\sqrt{-\frac{7}{4}} && \text{(SRP)}\\
q &= -\frac{1}{2} \pm \sqrt{-\frac{7}{4}} && \text{(CLA, BI)}\\
q &= -\frac{1}{2} \pm \frac{i\sqrt{7}}{2} && \text{(BI)}
\end{aligned}
$$

2.

$$
\begin{aligned}
y^2 + y &= 6 && \text{(given)}\\
y^2 + 2\cdot\frac{1}{2}y &= 6 && \text{(BI)}\\
y^2 + 2\cdot\frac{1}{2}y + \left(\frac{1}{2}\right)^2 &= 6 + \left(\frac{1}{2}\right)^2 && \text{(CLA)}\\
\left(y+\frac{1}{2}\right)^2 &= 6 + \left(\frac{1}{2}\right)^2 && \text{(PP2)}\\
\left(y+\frac{1}{2}\right)^2 &= \frac{25}{4} && \text{(Bi)}\\
y + \frac{1}{2} &= \pm\sqrt{\frac{25}{4}} && \text{(SRP)}\\
y &= -\frac{1}{2} \pm \sqrt{\frac{25}{4}} && \text{(CLA, BI)}\\
y &= -\frac{1}{2} \pm \frac{5}{2} && \text{(BI)}
\end{aligned}
$$

3.

$$
\begin{aligned}
x^2 - 6x &= 7 && \text{(given)} \\
x^2 + 2 \cdot -3x &= 7 && \text{(BI)} \\
x^2 + 2 \cdot -3x + (-3)^2 &= 7 + (-3)^2 && \text{(CLA)} \\
(x + -3)^2 &= 7 + (-3)^2 && \text{(PP2)} \\
(x + -3)^2 &= 16 && \text{(Bi)} \\
x + -3 &= \pm\sqrt{16} && \text{(SRP)} \\
x &= 3 \pm \sqrt{16} && \text{(CLA, BI)} \\
x &= 3 \pm 4 && \text{(BI)}
\end{aligned}
$$

4.

$$
\begin{aligned}
z^2 + 7z &= 4 && \text{(given)}\\
z^2 + 2 \cdot \frac{7}{2} z &= 4 && \text{(BI)}\\
z^2 + 2 \cdot \frac{7}{2} z + \left(\frac{7}{2}\right)^2 &= 4 + \left(\frac{7}{2}\right)^2 && \text{(CLA)}\\
\left(z + \frac{7}{2}\right)^2 &= 4 + \left(\frac{7}{2}\right)^2 && \text{(PP2)}\\
\left(z + \frac{7}{2}\right)^2 &= \frac{65}{4} && \text{(Bi)}\\
z + \frac{7}{2} &= \pm\sqrt{\frac{65}{4}} && \text{(SRP)}\\
z &= -\frac{7}{2} \pm \sqrt{\frac{65}{4}} && \text{(CLA, BI)}\\
z &= -\frac{7}{2} \pm \frac{\sqrt{65}}{2} && \text{(BI)}
\end{aligned}
$$

5.

$$
\begin{aligned}
w^2 - 3w &= 4 && \text{(given)} \\
w^2 + 2 \cdot -\frac{3}{2} w &= 4 && \text{(BI)} \\
w^2 + 2 \cdot -\frac{3}{2} w + \left(-\frac{3}{2}\right)^2 &= 4 + \left(-\frac{3}{2}\right)^2 && \text{(CLA)} \\
\left(w + -\frac{3}{2}\right)^2 &= 4 + \left(-\frac{3}{2}\right)^2 && \text{(PP2)} \\
\left(w + -\frac{3}{2}\right)^2 &= \frac{25}{4} && \text{(Bi)} \\
w + -\frac{3}{2} &= \pm\sqrt{\frac{25}{4}} && \text{(SRP)} \\
w &= \frac{3}{2} \pm \sqrt{\frac{25}{4}} && \text{(CLA, BI)} \\
w &= \frac{3}{2} \pm \frac{5}{2} && \text{(BI)}
\end{aligned}
$$

6.

$$w^2 - 3w = 5 \qquad \text{(given)}$$

$$w^2 + 2 \cdot -\frac{3}{2}w = 5 \qquad \text{(BI)}$$

$$w^2 + 2 \cdot -\frac{3}{2}w + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 \qquad \text{(CLA)}$$

$$\left(w + -\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 \qquad \text{(PP2)}$$

$$\left(w + -\frac{3}{2}\right)^2 = \frac{29}{4} \qquad \text{(Bi)}$$

$$w + -\frac{3}{2} = \pm\sqrt{\frac{29}{4}} \qquad \text{(SRP)}$$

$$w = \frac{3}{2} \pm \sqrt{\frac{29}{4}} \qquad \text{(CLA, BI)}$$

$$w = \frac{3}{2} \pm \frac{\sqrt{29}}{2} \qquad \text{(BI)}$$

7.

$$
\begin{aligned}
t^2 + 2t &= -3 && \text{(given)}\\
t^2 + 2\cdot 1t &= -3 && \text{(BI)}\\
t^2 + 2\cdot 1t + (1)^2 &= -3 + (1)^2 && \text{(CLA)}\\
(t+1)^2 &= -3 + (1)^2 && \text{(PP2)}\\
(t+1)^2 &= -2 && \text{(Bi)}\\
t+1 &= \pm\sqrt{-2} && \text{(SRP)}\\
t &= -1 \pm \sqrt{-2} && \text{(CLA, BI)}\\
t &= -1 \pm i\sqrt{2} && \text{(BI)}
\end{aligned}
$$

8.

$$\begin{aligned}
w^2 - 2w + 4 &= -w - 2 && \text{(given)}\\
w^2 - w &= -6 && \text{(BI)}\\
w^2 + 2 \cdot -\frac{1}{2}w &= -6 && \text{(BI)}\\
w^2 + 2 \cdot -\frac{1}{2}w + \left(-\frac{1}{2}\right)^2 &= -6 + \left(-\frac{1}{2}\right)^2 && \text{(CLA)}\\
\left(w + -\frac{1}{2}\right)^2 &= -6 + \left(-\frac{1}{2}\right)^2 && \text{(PP2)}\\
\left(w + -\frac{1}{2}\right)^2 &= -\frac{23}{4} && \text{(Bi)}\\
w + -\frac{1}{2} &= \pm\sqrt{-\frac{23}{4}} && \text{(SRP)}\\
w &= \frac{1}{2} \pm \sqrt{-\frac{23}{4}} && \text{(CLA, BI)}\\
w &= \frac{1}{2} \pm \frac{i\sqrt{23}}{2} && \text{(BI)}
\end{aligned}$$

9.

$$\begin{aligned}
r^2 - 12r - 6 &= 1 - 6r && \text{(given)}\\
r^2 - 6r &= 7 && \text{(BI)}\\
r^2 + 2 \cdot -3r &= 7 && \text{(BI)}\\
r^2 + 2 \cdot -3r + (-3)^2 &= 7 + (-3)^2 && \text{(CLA)}\\
(r + -3)^2 &= 7 + (-3)^2 && \text{(PP2)}\\
(r + -3)^2 &= 16 && \text{(Bi)}\\
r + -3 &= \pm\sqrt{16} && \text{(SRP)}\\
r &= 3 \pm \sqrt{16} && \text{(CLA, BI)}\\
r &= 3 \pm 4 && \text{(BI)}
\end{aligned}$$

10.

$$\begin{aligned}
u^2 - 8u - 4 &= 3 - 4u && \text{(given)}\\
u^2 - 4u &= 7 && \text{(BI)}\\
u^2 + 2 \cdot -2u &= 7 && \text{(BI)}\\
u^2 + 2 \cdot -2u + (-2)^2 &= 7 + (-2)^2 && \text{(CLA)}\\
(u + -2)^2 &= 7 + (-2)^2 && \text{(PP2)}\\
(u + -2)^2 &= 11 && \text{(Bi)}\\
u + -2 &= \pm\sqrt{11} && \text{(SRP)}\\
u &= 2 \pm \sqrt{11} && \text{(CLA, BI)}\\
u &= 2 \pm \sqrt{11} && \text{(BI)}
\end{aligned}$$

11.

$$
\begin{aligned}
t^2 + 10t - 3 &= 5t + 1 && \text{(given)}\\
t^2 + 5t &= 4 && \text{(BI)}\\
t^2 + 2 \cdot \frac{5}{2} t &= 4 && \text{(BI)}\\
t^2 + 2 \cdot \frac{5}{2} t + \left(\frac{5}{2}\right)^2 &= 4 + \left(\frac{5}{2}\right)^2 && \text{(CLA)}\\
\left(t + \frac{5}{2}\right)^2 &= 4 + \left(\frac{5}{2}\right)^2 && \text{(PP2)}\\
\left(t + \frac{5}{2}\right)^2 &= \frac{41}{4} && \text{(Bi)}\\
t + \frac{5}{2} &= \pm\sqrt{\frac{41}{4}} && \text{(SRP)}\\
t &= -\frac{5}{2} \pm \sqrt{\frac{41}{4}} && \text{(CLA, BI)}\\
t &= -\frac{5}{2} \pm \frac{\sqrt{41}}{2} && \text{(BI)}
\end{aligned}
$$

12.

$$\begin{aligned}
27v - 3v^2 &= -6 && \text{(given)}\\
-\frac{1}{3}\left(27v - 3v^2\right) &= -\frac{1}{3}\left(-6\right) && \text{(CLM)}\\
v^2 - 9v &= 2 && \text{(BI)}\\
v^2 + 2 \cdot -\frac{9}{2}v &= 2 && \text{(BI)}\\
v^2 + 2 \cdot -\frac{9}{2}v + \left(-\frac{9}{2}\right)^2 &= 2 + \left(-\frac{9}{2}\right)^2 && \text{(CLA)}\\
\left(v + -\frac{9}{2}\right)^2 &= 2 + \left(-\frac{9}{2}\right)^2 && \text{(PP2)}\\
\left(v + -\frac{9}{2}\right)^2 &= \frac{89}{4} && \text{(Bi)}\\
v + -\frac{9}{2} &= \pm\sqrt{\frac{89}{4}} && \text{(SRP)}\\
v &= \frac{9}{2} \pm \sqrt{\frac{89}{4}} && \text{(CLA, BI)}\\
v &= \frac{9}{2} \pm \frac{\sqrt{89}}{2} && \text{(BI)}
\end{aligned}$$

13.

$$
\begin{aligned}
3u^2 - 18u &= -24 && \text{(given)} \\
\frac{1}{3}\left(3u^2 - 18u\right) &= \frac{1}{3}(-24) && \text{(CLM)} \\
u^2 - 6u &= -8 && \text{(BI)} \\
u^2 + 2 \cdot -3u &= -8 && \text{(BI)} \\
u^2 + 2 \cdot -3u + (-3)^2 &= -8 + (-3)^2 && \text{(CLA)} \\
(u + -3)^2 &= -8 + (-3)^2 && \text{(PP2)} \\
(u + -3)^2 &= 1 && \text{(Bi)} \\
u + -3 &= \pm\sqrt{1} && \text{(SRP)} \\
u &= 3 \pm \sqrt{1} && \text{(CLA, BI)} \\
u &= 3 \pm 1 && \text{(BI)}
\end{aligned}
$$

14.

$$
\begin{aligned}
-3q^2 - 30q &= -15 && \text{(given)}\\
-\frac{1}{3}\left(-3q^2 - 30q\right) &= -\frac{1}{3}(-15) && \text{(CLM)}\\
q^2 + 10q &= 5 && \text{(BI)}\\
q^2 + 2\cdot 5q &= 5 && \text{(BI)}\\
q^2 + 2\cdot 5q + (5)^2 &= 5 + (5)^2 && \text{(CLA)}\\
(q+5)^2 &= 5 + (5)^2 && \text{(PP2)}\\
(q+5)^2 &= 30 && \text{(Bi)}\\
q + 5 &= \pm\sqrt{30} && \text{(SRP)}\\
q &= -5 \pm \sqrt{30} && \text{(CLA, BI)}\\
q &= -5 \pm \sqrt{30} && \text{(BI)}
\end{aligned}
$$

15.

$$-3t^2 - 24t = -30 \qquad \text{(given)}$$
$$-\frac{1}{3}\left(-3t^2 - 24t\right) = -\frac{1}{3}(-30) \qquad \text{(CLM)}$$
$$t^2 + 8t = 10 \qquad \text{(BI)}$$
$$t^2 + 2 \cdot 4t = 10 \qquad \text{(BI)}$$
$$t^2 + 2 \cdot 4t + (4)^2 = 10 + (4)^2 \qquad \text{(CLA)}$$
$$(t+4)^2 = 10 + (4)^2 \qquad \text{(PP2)}$$
$$(t+4)^2 = 26 \qquad \text{(Bi)}$$
$$t + 4 = \pm\sqrt{26} \qquad \text{(SRP)}$$
$$t = -4 \pm \sqrt{26} \qquad \text{(CLA, BI)}$$
$$t = -4 \pm \sqrt{26} \qquad \text{(BI)}$$

Section 0.6

1.

$$3z^2 + 6z - 5 = 0 \qquad \text{(given)}$$
$$z = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 3 \cdot -5}}{2 \cdot 3} \qquad \text{(QF)}$$
$$z = \frac{1}{6}\left(-6 \pm 4\sqrt{6}\right) \qquad \text{(BI)}$$

2.

$$3r^2 + 7r + 3 = 0 \qquad \text{(given)}$$
$$r = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3} \qquad \text{(QF)}$$
$$r = \frac{1}{6}\left(-7 \pm \sqrt{13}\right) \qquad \text{(BI)}$$

3.

$$2r^2 + 4r - 7 = 0 \qquad \text{(given)}$$

$$r = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot -7}}{2 \cdot 2} \qquad \text{(QF)}$$

$$r = \frac{1}{4}\left(-4 \pm 6\sqrt{2}\right) \qquad \text{(BI)}$$

4.

$$r^2 + 5r - 1 = 0 \qquad \text{(given)}$$

$$r = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot -1}}{2 \cdot 1} \qquad \text{(QF)}$$

$$r = \frac{1}{2}\left(-5 \pm \sqrt{29}\right) \qquad \text{(BI)}$$

5.

$$2y^2 - 6y + 7 = 0 \qquad \text{(given)}$$

$$y = \frac{6 \pm \sqrt{-6^2 - 4 \cdot 2 \cdot 7}}{2 \cdot 2} \qquad \text{(QF)}$$

$$y = \frac{1}{4}\left(6 \pm 2i\sqrt{5}\right) \qquad \text{(BI)}$$

6.

$$2u^2 - 2u + 5 = 0 \qquad \text{(given)}$$

$$u = \frac{2 \pm \sqrt{-2^2 - 4 \cdot 2 \cdot 5}}{2 \cdot 2} \qquad \text{(QF)}$$

$$u = \frac{1}{4}(2 \pm 6i) \qquad \text{(BI)}$$

7.

$$2y^2 + 5y - 3 = 0 \qquad \text{(given)}$$

$$y = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot -3}}{2 \cdot 2} \qquad \text{(QF)}$$

$$y = \frac{1}{4}(-5 \pm 7) \qquad \text{(BI)}$$

8.

$$\begin{aligned} 3v^2 - 4v - 2 &= 0 && \text{(given)} \\ v &= \frac{4 \pm \sqrt{-4^2 - 4 \cdot 3 \cdot -2}}{2 \cdot 3} && \text{(QF)} \\ v &= \frac{1}{6}\left(4 \pm 2\sqrt{10}\right) && \text{(BI)} \end{aligned}$$

9.

$$\begin{aligned} 2w^2 + 5w - 2 &= 0 && \text{(given)} \\ w &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot -2}}{2 \cdot 2} && \text{(QF)} \\ w &= \frac{1}{4}\left(-5 \pm \sqrt{41}\right) && \text{(BI)} \end{aligned}$$

10.

$$\begin{aligned} 2r^2 - 5r + 2 &= 0 && \text{(given)} \\ r &= \frac{5 \pm \sqrt{-5^2 - 4 \cdot 2 \cdot 2}}{2 \cdot 2} && \text{(QF)} \\ r &= \frac{5 \pm 3}{4} && \text{(BI)} \end{aligned}$$

Section 0.7

1.

$$\begin{aligned} & |9y + 7| = 13 && \text{(given)} \\ 9y + 7 = 13 \quad & \text{OR} \quad 9y + 7 = -(13) && \text{(AVT)} \\ y = \frac{2}{3} \quad & \text{OR} \quad y = -\frac{20}{9} && \text{(BI)} \end{aligned}$$

after checking each one we see the solution set it: $\left\{-\frac{20}{9}, \frac{2}{3}\right\}$

2.

$$\begin{aligned} & |4w + 9| = 9 && \text{(given)} \\ 4w + 9 = 9 \quad & \text{OR} \quad 4w + 9 = -(9) && \text{(AVT)} \\ w = 0 \quad & \text{OR} \quad w = -\frac{9}{2} && \text{(BI)} \end{aligned}$$

after checking each one we see the solution set it: $\left\{-\frac{9}{2}, 0\right\}$

3.

$$\begin{aligned} &|6v+19| = 9 && \text{(given)}\\ 6v+19=9 \qquad \text{OR} \qquad & 6v+19 = -(9) && \text{(AVT)}\\ v = -\frac{5}{3} \qquad \text{OR} \qquad & v = -\frac{14}{3} && \text{(BI)} \end{aligned}$$

after checking each one we see the solution set it: $\left\{-\frac{14}{3}, -\frac{5}{3}\right\}$

4.

$$\begin{aligned} &|6z+6| = 2 && \text{(given)}\\ 6z+6=2 \qquad \text{OR} \qquad & 6z+6 = -(2) && \text{(AVT)}\\ z = -\frac{2}{3} \qquad \text{OR} \qquad & z = -\frac{4}{3} && \text{(BI)} \end{aligned}$$

after checking each one we see the solution set it: $\left\{-\frac{4}{3}, -\frac{2}{3}\right\}$

5.

$$\begin{aligned} &|5q| = 1 && \text{(given)}\\ 5q=1 \qquad \text{OR} \qquad & 5q = -(1) && \text{(AVT)}\\ q = \frac{1}{5} \qquad \text{OR} \qquad & q = -\frac{1}{5} && \text{(BI)} \end{aligned}$$

after checking each one we see the solution set it: $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$

6.

$$\begin{aligned} &\left|\frac{z}{2}+4\right| = 3-4z && \text{(given)}\\ \frac{z}{2}+4=3-4z \qquad \text{OR} \qquad & \frac{z}{2}+4 = -(3-4z) && \text{(AVT)}\\ z = -\frac{2}{9} \qquad \text{OR} \qquad & z = 2 && \text{(BI)} \end{aligned}$$

after checking each one we see the solution set it: $\left\{-\frac{2}{9}\right\}$

7.

$$\begin{aligned} \frac{1}{2v} &= \frac{1}{6v^2} + \frac{1}{6v} && \text{(given)}\\ {\color{blue}12v^2}\left(\frac{1}{2v}\right) &= {\color{blue}12v^2}\left(\frac{1}{6v^2} + \frac{1}{6v}\right) && \text{(CLM)}\\ 6v &= 2v+2 && \text{(DL, BI)}\\ v &= \frac{1}{2} && \text{(BI)} \end{aligned}$$

plug and check, $v = \frac{1}{2}$

8.

$$\begin{aligned} \frac{7}{r}+1 &= \frac{1}{r}+5 && \text{(given)} \\ {\color{blue}r}\left(\frac{7}{r}+1\right) &= {\color{blue}r}\left(\frac{1}{r}+5\right) && \text{(CLM)} \\ r+7 &= 5r+1 && \text{(DL, BI)} \\ r &= \frac{3}{2} && \text{(BI)} \end{aligned}$$

plug and check, $r = \frac{3}{2}$

9.

$$\begin{aligned} \frac{1}{7x}+\frac{1}{5} &= \frac{1}{7x}+\frac{1}{35} && \text{(given)} \\ {\color{blue}35x}\left(\frac{1}{7x}+\frac{1}{5}\right) &= {\color{blue}35x}\left(\frac{1}{7x}+\frac{1}{35}\right) && \text{(CLM)} \\ 7x+5 &= x+5 && \text{(DL, BI)} \\ x &= 0 && \text{(BI)} \end{aligned}$$

plug and check, plug-n-check $\implies$ no real solution

10.

$$\begin{aligned} \frac{1}{2r}+\frac{1}{4} &= \frac{1}{3r} && \text{(given)} \\ {\color{blue}12r}\left(\frac{1}{2r}+\frac{1}{4}\right) &= {\color{blue}12r}\left(\frac{1}{3r}\right) && \text{(CLM)} \\ 3r+6 &= 4 && \text{(DL, BI)} \\ r &= -\frac{2}{3} && \text{(BI)} \end{aligned}$$

plug and check, $r = -\frac{2}{3}$

11.

$$\begin{aligned} 5x+3y &= 67 && \text{(given EQ:A)} \\ x &= \frac{67}{5}-\frac{3y}{5} \end{aligned}$$

$$\begin{aligned} 5x+2y &= 58 && \text{(given EQ:B)} \\ 5{\color{blue}\left(\frac{67}{5}-\frac{3y}{5}\right)}+2y &= 58 && \text{(sub)} \\ y &= 9 && \text{(BI)} \end{aligned}$$

Plug this into either equation to get $(x, y) = (8, 9)$

12.

$$\begin{aligned} 4x + y &= 6 && \text{(given EQ:A)} \\ x &= \frac{3}{2} - \frac{y}{4} \end{aligned}$$

$$\begin{aligned} x + 3y &= 7 && \text{(given EQ:B)} \\ \left(\frac{3}{2} - \frac{y}{4}\right) + 3y &= 7 && \text{(sub)} \\ y &= 2 && \text{(BI)} \end{aligned}$$

Plug this into either equation to get $(x, y) = (1, 2)$

13.

$$\begin{aligned} -2x - 2y &= -18 && \text{(given EQ:A)} \\ x &= 9 - y \end{aligned}$$

$$\begin{aligned} -3x - 5y &= -47 && \text{(given EQ:B)} \\ -3(9 - y) - 5y &= -47 && \text{(sub)} \\ y &= 10 && \text{(BI)} \end{aligned}$$

Plug this into either equation to get $(x, y) = (-1, 10)$

14.

$$\begin{aligned} -5x - y &= -44 && \text{(given EQ:A)} \\ x &= \frac{44}{5} - \frac{y}{5} \end{aligned}$$

$$\begin{aligned} -2x - y &= -17 && \text{(given EQ:B)} \\ -2\left(\frac{44}{5} - \frac{y}{5}\right) - y &= -17 && \text{(sub)} \\ y &= -1 && \text{(BI)} \end{aligned}$$

Plug this into either equation to get $(x, y) = (9, -1)$

15.

$$\begin{aligned} 4x + y &= 35 && \text{(given EQ:A)} \\ x &= \frac{35}{4} - \frac{y}{4} \end{aligned}$$

$$\begin{aligned} 4x + 4y &= 32 && \text{(given EQ:B)} \\ 4\left(\frac{35}{4} - \frac{y}{4}\right) + 4y &= 32 && \text{(sub)} \\ y &= -1 && \text{(BI)} \end{aligned}$$

Plug this into either equation to get $(x, y) = (9, -1)$

Section 0.8

1. At $t = 10$ we substitute to get

$$h = -15(10)^2 + 300(10) + 350 = 1850ft.$$

At $t = 13$ we get

$$h = -15(13)^2 + 300(13) + 350 = 1715ft.$$

At $t = 19$ we get

$$h = -15(19)^2 + 300(19) + 350 = 635ft.$$

2. At $t = 1$ we substitute to get

$$h = -10(1)^2 + 120(1) + 120 = 230ft.$$

At $t = 5$ we get

$$h = -10(5)^2 + 120(5) + 120 = 470ft.$$

At $t = 8$ we get

$$h = -10(8)^2 + 120(8) + 120 = 440ft.$$

3. 'hits the ground' means $h = 0$ So we substitute

$$0 = -14t^2 + 40t + 300$$

and solve:

$$\begin{aligned} -14t^2 + 40t + 300 &= h && \text{(given)} \\ -14t^2 + 40t + 300 &= 0 && \text{(sub, gound}=h=0\text{)} \\ t = \frac{-(40) \pm \sqrt{(40)^2 - 4(\text{-}14)(300)}}{2(\text{-}14)} & && \text{(QF)} \\ t \approx -3.4159 \qquad \text{OR} \qquad t &\approx 6.2731 && \text{(calculator)} \end{aligned}$$

assume the t is positive... so the object hits the ground at $t \approx 6.2731$ seconds!

4. We use pythagoras!

$$\begin{aligned} (30)^2 &= (4)^2 + (u)^2 && \text{(given)} \\ u &= \pm\sqrt{884} && \text{(CLA, SRP,bi)} \\ u &\approx \pm 29.73 && \text{(calc)} \end{aligned}$$

assume the u is positive, $u \approx 29.73$

5. We use pythagoras!

$$\begin{aligned}(60)^2 &= (3)^2 + (w)^2 && \text{(given)}\\ w &= \pm\sqrt{3591} && \text{(CLA, SRP,bi)}\\ w &\approx \pm\, 59.92 && \text{(calc)}\end{aligned}$$

assume the w is positive, $w \approx 59.92$

6. We use pythagoras!

$$\begin{aligned}(90)^2 &= (t)^2 + (3)^2 && \text{(given)}\\ t &= \pm\sqrt{8091} && \text{(CLA, SRP,bi)}\\ t &\approx \pm\, 89.95 && \text{(calc)}\end{aligned}$$

assume the t is positive, $t \approx 89.95$

7. We use pythagoras!

$$\begin{aligned}(15)^2 &= (3)^2 + (w)^2 && \text{(given)}\\ w &= \pm\sqrt{216} && \text{(CLA, SRP,bi)}\\ w &\approx \pm\, 14.70 && \text{(calc)}\end{aligned}$$

assume the w is positive, $w \approx 14.70$

8. First convert to miles, each foot is worth 0.000189393939 miles, so we simply muliply to get the height to be $1050 \times 0.000189393939 miles = 0.198864\, miles$ then use pythagoras.

$$\begin{aligned}(3963.2)^2 &= (3963)^2 + (x)^2 && \text{(given)}\\ x &= \pm\sqrt{1576.23} && \text{(CLA, SRP,bi)}\\ x &\approx \pm\, 39.7018 && \text{(calc)}\end{aligned}$$

assume the x is positive, $x \approx 39.7018$ miles!

9. First convert to miles, each foot is worth 0.000189393939 miles, so we simply muliply to get the height to be $480 \times 0.000189393939 miles = 0.0909091\, miles$ then use pythagoras.

$$\begin{aligned}(3963.09)^2 &= (3963)^2 + (x)^2 && \text{(given)}\\ x &= \pm\sqrt{720.554} && \text{(CLA, SRP,bi)}\\ x &\approx \pm\, 26.8431 && \text{(calc)}\end{aligned}$$

assume the x is positive, $x \approx 26.8431$ miles!

10. First convert to miles, each foot is worth 0.000189393939 miles, so we simply muliply to get the height to be $520 \times 0.000189393939 miles = 0.0984848\, miles$ then use pythagoras.

$$\begin{aligned}(3963.1)^2 &= (3963)^2 + (x)^2 && \text{(given)}\\ x &= \pm\sqrt{780.601} && \text{(CLA, SRP,bi)}\\ x &\approx \pm\, 27.9392 && \text{(calc)}\end{aligned}$$

assume the x is positive, $x \approx 27.9392$ miles!

11. First convert to miles, each foot is worth 0.000189393939 miles, so we simply muliply to get the height to be $1050 \times 0.000189393939 miles = 0.198864\, miles$ then use pythagoras.

$$\begin{aligned} (3963.2)^2 &= (3963)^2 + (x)^2 && \text{(given)} \\ x &= \pm\sqrt{1576.23} && \text{(CLA, SRP,bi)} \\ x &\approx \pm\, 39.7018 && \text{(calc)} \end{aligned}$$

assume the x is positive, $x \approx 39.7018$ miles!

12. solve the system of equations $\mathrm{p} + \mathrm{q} = 28$ and $\mathrm{p} + 25\mathrm{q} = 484.$, the answer is $\mathrm{p} = 9$ and $\mathrm{q} = 19$

13. solve the system of equations $\mathrm{q} + \mathrm{n} = 32$ and $25\mathrm{q} + 5\mathrm{n} = 500.$, the answer is $\mathrm{q} = 17$ and $\mathrm{n} = 15$

14. solve the system of equations $\mathrm{h} + \mathrm{n} = 28$ and $50\mathrm{h} + 5\mathrm{n} = 770.$, the answer is $\mathrm{h} = 14$ and $\mathrm{n} = 14$

15. solve the system of equations $\mathrm{n} + \mathrm{p} = 27$ and $5\mathrm{n} + \mathrm{p} = 83.0$, the answer is $\mathrm{n} = 14$ and $\mathrm{p} = 13$

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