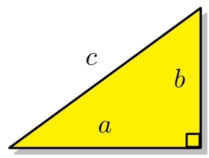
CONTENTS

0.1 Pythagorean Theorem

We introduced this theorem in the late part of Chapter 5, but it hardly needs introduction. It is an icon of human intellect and grace. While we used it in chapter 5 as a way of practicing our solving skills, here we take the opportunity to wonder why it is so. Again, the statement of the theorem is:

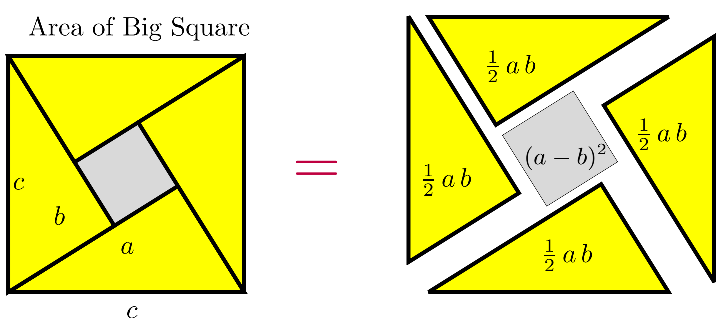
Theorem 0.1.1. *The Pythagorean Theorem [PT] [also known as Pythagoras Theorem] says if a triangle has sides of real values, a, b, and c where c is the hypotenuse, then*

$$c^2 = a^2 + b^2$$



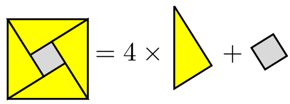
If you are now wondering *why?* you are not alone. Humans from every walk of life, of every age, every color, from every continent, from have wondered *why* for more than 4,000 years. More than wonder, people have resolved the ”why” in beautiful proofs of all flavors employing a very wide range of creative ideas that work to prove the theorem. This theorem has become *the most proven* theorem of all times. Inmates, priests, scientists, USA presidents, and drunkards have all designed their own beautiful proofs. There are entire books dedicated to exploring these [4].

Here we present a standard and now classic proof. Essentially the proof is involves making a large square composed of 4 triangles and a small square, then the brilliant breakthrough: to note that the area of the entire square is equal to the sum of the areas of the little pieces!!



To complete the proof we need only make the observation that the large square has the same areas as the sum of the smaller areas. It should be said that we are assuming all of the triangles on the diagram are identical and all are right triangles.

$$BigSquareArea = LittlePiecesArea$$

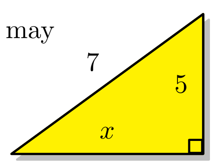


$$c^2 = 4\left(\frac{1}{2} \cdot a \cdot b\right) + (a-b)^2$$
$$c^2 = 2ab + (a-b)(a-b) \tag{BI}$$
$$c^2 = 2ab + a^2 - 2ab + b^2 \tag{FOIL, BI}$$
$$c^2 = a^2 + b^2 \tag{QED}$$

Look for other amazingly creative proofs, including one by the 20th President, James A. Garfield. Some of these proofs are included in the corresponding lecture, some are in the homework and some in the self quizzes. For now, we turn to our attention to elsewhere and see how the Pythagorean Theorem fits with the rest of the universe.

The Pythagoras Theorem may be used to solve the missing side of a right triangle. This works particularly well when two of the three sides are known. When two sides are known Pythagoras will elegantly deliver the third side for us. Moreover, the reader should find this type of example a review, as we previously used when we first introduced Pythagoras and we wanted to practice our degree two solving skills.

Example Use PT to find the missing side. You may assume the side is of positive length

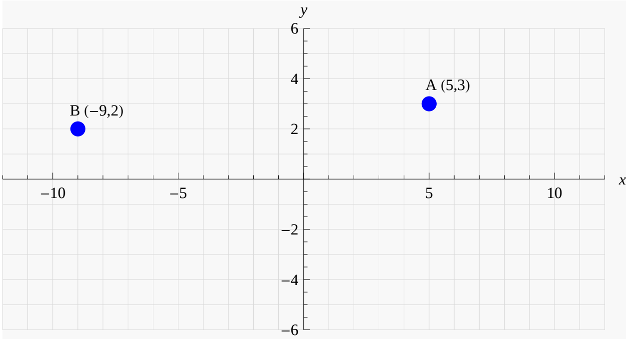


Solution:

$$7^2 = x^2 + 5^2 \tag{PT}$$
$$49 = x^2 + 25 \tag{BI}$$
$$24 = x^2 \tag{CLA, BI}$$
$$\pm\sqrt{24} = x \tag{SRP}$$

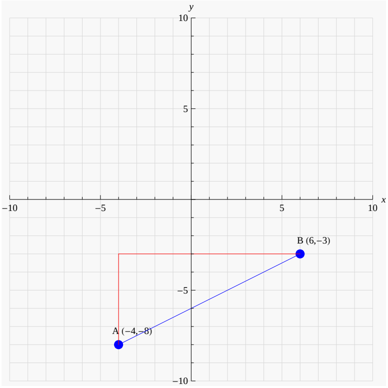
Assuming the side is positive, $x = \sqrt{24}$.

Now we subtly introduce the Cartesian plane while at the same time we find another excellent use of Pythagoras, namely to compute distances between two points. First a nano-introduction to the Cartesian plane. Such plane is a collection of points, sometimes called ordered pairs. They are usually expressed with parenthesis and a comma separating them such as the point $(3, 5)$. The Cartesian plane is usually a horizontal number line called the x axis, together with a vertical line called the y-axis. The points, or ordered pairs can be placed on the plane by first locating the horizontal position of the x point, the first or the numbers, then from there locating the vertical position of the second coordinate, the y-coordinate. Here are a couple of example showing a couple points and their respective positions.



Having introduced the Cartesian coordinate system, we now show how the distance between two points can be computed.

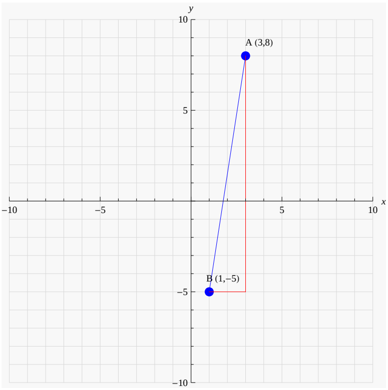
Example Compute the distance between the two points $(-4, -8)$ and $(6, -3)$.



Solution: The key is to draw a triangle, turning the sought distance into a hypotenuse or a right triangle, then we can use PT. We count the squares to obtain the size of each of the sides, then...

$$d^2 = 5^2 + 10^2 \qquad \text{(PT)}$$
$$d = \pm\sqrt{125} \qquad \text{(Bi)}$$

Example This time compute the distance between the two points $(1, -5)$ and $(3, 8)$.



Solution: Again, the key is to draw a triangle, turning the sought distance into a hypotenuse or a right triangle, then we can use PT. We count the squares to obtain the size of each of the sides, then...

$$d^2 = 2^2 + 13^2 \qquad \text{(PT)}$$
$$d^2 = 4 + 169 \qquad \text{(PT)}$$
$$d = \pm\sqrt{173} \qquad \text{(SRP, Bi)}$$

Here goes an important observation. In the previous example, counting the vertical distance may be a bit insect-ish. Perhaps a more graceful way of counting the vertical distance between $(1, -5)$ and $(3, 8)$ is to take the difference between the top value and the lower one, thus we see the vertical distance can be computed as

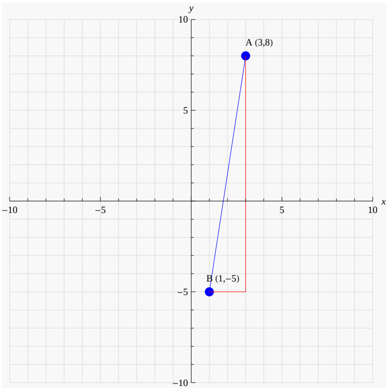
$$Vertical\ Distance = 8 - (-5) = 13$$

Similarly we can compute the horizontal distance by checking the difference between the corresponding x coordinates in $(1, -5)$ and $(3, 8)$ as in

$$Horizontal\ Distance = 3 - (1) = 2$$

Such observation may help us avoid the insect-like task of counting squares to compute the distance, thus the same example may be done like this:

Example This time compute the distance between the two points $(1, -5)$ and $(3, 8)$.

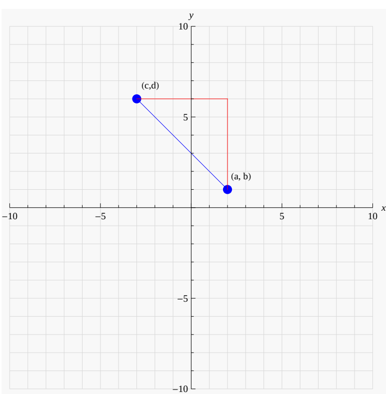


Solution: Again, the key is to draw a triangle, turning the sought distance into a hypotenuse or a right triangle, then we can use PT. This time we DO NOT count, we subtract coordinates.

$$d^2 = [3 - 1]^2 + [8 - {}^{-}5]^2 \quad \text{(PT)}$$
$$d^2 = 2^2 + 13^2 \quad \text{(PT)}$$
$$d^2 = 4 + 169 \quad \text{(PT)}$$
$$d = \pm\sqrt{173} \quad \text{(SRP, Bi)}$$

The previous version of the example is very important because it serves as a stepping stone to create a generic template formula which could be used on any pair of point to compute the distance. Consider the following example, it is a generic template.

Example This time compute the distance between the two points (a, b) and (c, d).



Solution: Again, the key is to draw a triangle, turning the sought distance into a hypotenuse or a right triangle, then we can use PT. This time we CAN NOT count, but we CAN subtract coordinates.

$$d^2 = [c - a]^2 + [d - b]^2 \quad \text{(PT)}$$
$$d = \pm\sqrt{[c - a]^2 + [d - b]^2} \quad \text{(SRP, Bi)}$$

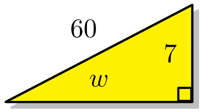
The previous result is important and common enough that we endow it with theorem status.

Theorem 0.1.2. *The Distance Formula says if (a, b) and (c, d) are distinct points on the Cartesian plane then the distance between, d, them is given by*

$$d = \sqrt{[c - a]^2 + [d - b]^2}$$

0.1.1 Exercises

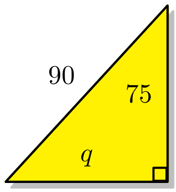
1. Determine the missing side of the triangle [not drawn to scale].



2. Plot the following points: $A(7,-9)$ and $B(4,7)$ on a Cartesian Plane.

3. Plot the following points: $A(-6,8)$ and $B(-3,6)$ on a Cartesian Plane.

4. Determine the missing side of the triangle [not drawn to scale].



5. Find the distance between points: $A(3,7)$ and $B(-8,-9)$

6. Find the distance between points: $A(-4,-3)$ and $B(-7,-4)$

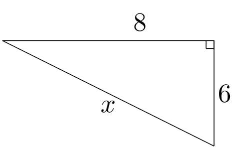
7. Find the distance between points: $A(9,1)$ and $B(-1,-3)$

8. Find the distance between points: $A(-4,1)$ and $B(-7,6)$

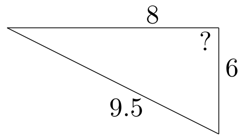
9. Find the distance between points: $(-V,-X)$ and (U,ρ)

10. Find the distance between points: $(-\beta,U)$ and $(\rho,-\gamma)$

11. **Straight Walls?** Suppose you are planning to build a rectangular room 8ft. by 6ft., In theory, how long should the diagonal measurement be? solve for x

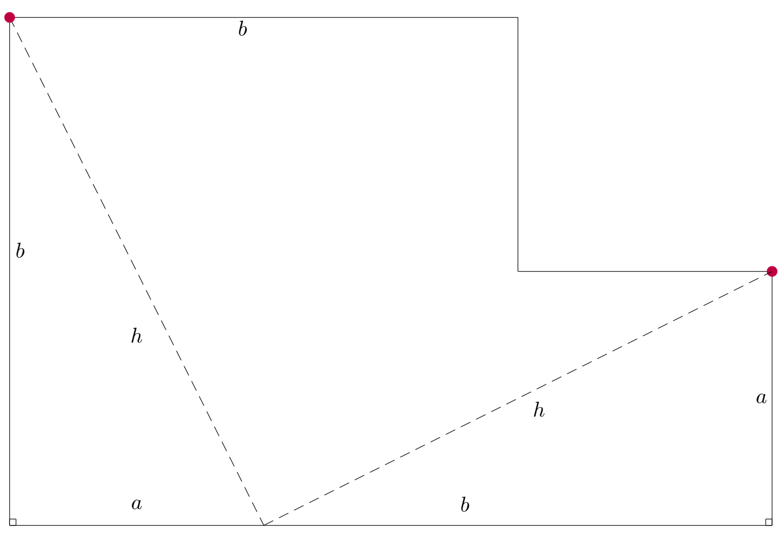


12. Got Straight Walls? Suppose when you are done building the first two walls shown below, you check the diagonal and it measures 9.5 ft. What can you say about the angle between the walls. Less or more than 90°?

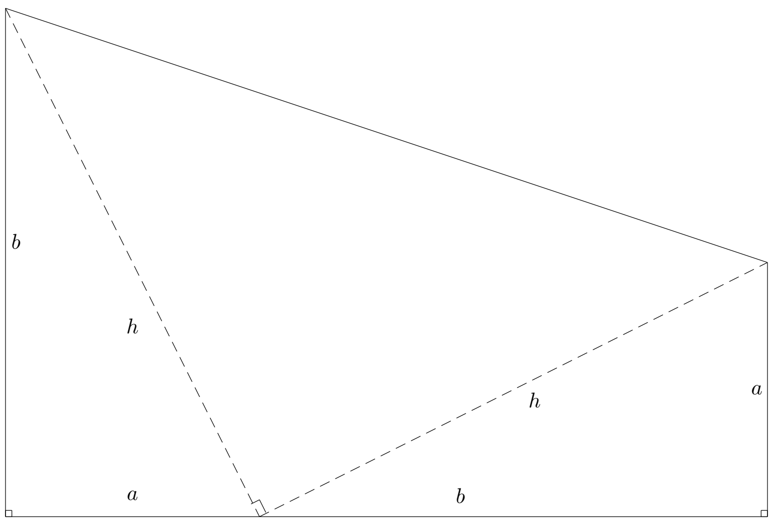


13. Do you think this theorem is 'reversible' that is, if $a^2 + b^2 = h^2$ does that mean we have a right triangle for sure?

14. Another Proof of Pythagoras Thm. Find the area of the figure below. Note its is made up of two squares, one b by b, the other measures, a by a. Then, use scissors to cut out the figure below. After cutting the figure, cut the dashed lines. Can you make a perfect square out of the 3 cut figures? What is the area of the square? What does this show?

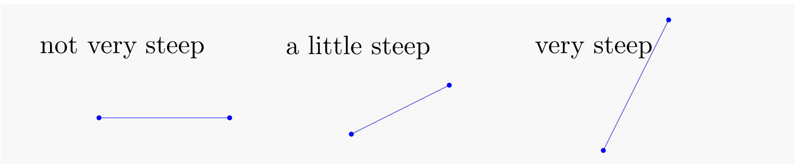


15. Proof by US President, James Garfield (1876) You know what to do (compare the area of the trapezoid to the sum of the little areas). Just keep in mind: area of a trapezoid is width, $a + b$ in this case, times the average of the heights, $(a + b)/2$.



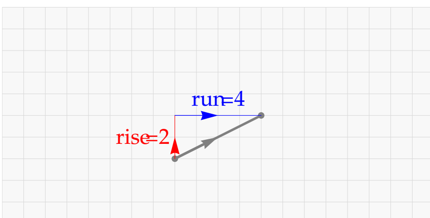
0.2 The Slope

For the next couple sections we turn our attention to lines, their graphs, and their equations. Before we get too far into the details we take the opportunity here to get conformable with the notion of *slope of a line*. It turns out that one of the key characteristics of a line will turn out be its 'steepness'. While we may speak of terms like 'steep', 'not too steep', or 'very steep', it may be helpful to actually put a number of measurement on the steepness. For example,



Again, what we would like is to have a way of measuring 'steepness' in a more precise way. Traditionally, This has been done using a number called *the slope* of the line. Roughly speaking, we compute this number by first placing a grid on on the background, then we compute the vertical distance [the 'rise'] as compared to the horizontal distance [the 'run']. This proportion of vertical distance vs. horizontal distance [rise/over/run] is what we define as the *slope* of the line.

Example

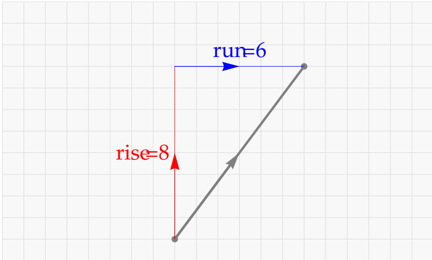


Thus we would say that the slope of such line is

$$slope = \frac{rise}{run} = \frac{2}{4}$$

The variable m is commonly used to represent the slope. Often we may simplify this to say $m = \frac{2}{4} = \frac{1}{2} = .50$. On highways it is also common to describe the slope as the 'grade' in this case the grade [aka. Gradient, Slope, Grade, Pitch, Rise Over Run Ratio] $m = .50 = 50\%$

Example

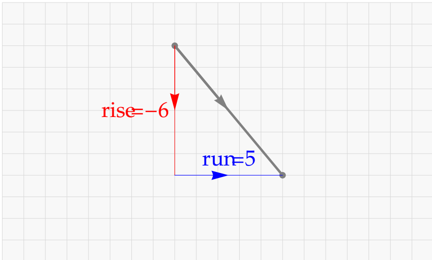


In this case the slope is

$$slope = \frac{rise}{run} = \frac{8}{6} = \frac{4}{3}$$

Example

We can also have negative slopes. These occur on 'downhills'.



In this case the slope is

$$slope = \frac{rise}{run} = \frac{-6}{5}$$

There are some special cases. For example, when the line is 'flat' we say the slope is zero. When the line is completely vertical, we say the slope is not real, or infinite [$m = \infty$]. Moreover, rather than computing the slope from looking at the graph, we could compute the slope just from knowing two distinct points on the line.

Example

Find the slope of the line that goes through $(2,4)$ and $(5,10)$

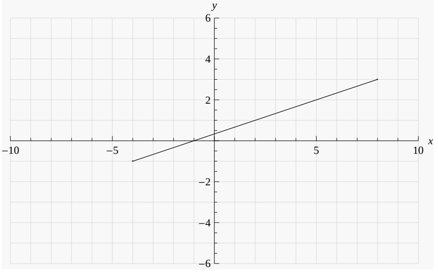
Solution:

We first compute the rise. To do this, we look at the y coordinates and compute the difference. From $(2,4)$ to $(5,10)$, the rise is $rise = 10-4 = 6$, then we compute the run from $(2,4)$ to $(5,10)$, $run = 5-2 = 3$. Therefore, the slope is given by

$$slope = \frac{rise}{run} = \frac{10-4}{5-2} = \frac{6}{3} = 2$$

Finally, we present our last example. In this case, we graph a line and ask for the calculation of the slope. It is important to experiment freely and see that the steepness is the same as measured from *any* pair of distinct point lying on the line.

Example Find the slope of the line.



The first thing to do here is select [any] two distinct point on the line. Based on the graph, we will assume it goes through $(2,1)$ and $(4,2)$. We then compute the slope as:

$$slope = \frac{rise}{run} = \frac{2-1}{4-2} = \frac{1}{2}$$

We conclude this section with a formal definition of the slope.

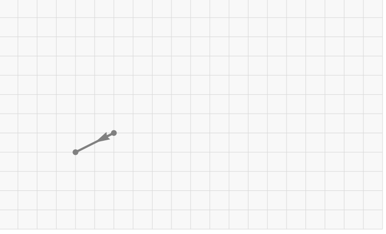
Definition Suppose a,b,c,d are real numbers. We define the *slope* from point (a,b) to (c,d) as follows. If a and c are equal we say the slope is infinite. Other wise, we define the slope, m as

$$m = \frac{d-b}{c-a}$$

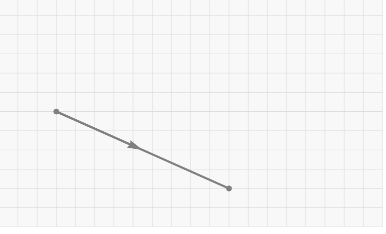
0.2.1 Exercises

1. Compute the slope of the segment

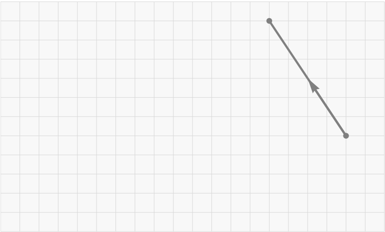
2. Compute the slope of the segment



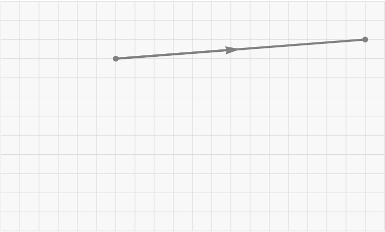
3. Compute the slope of the segment



4. Compute the slope of the segment



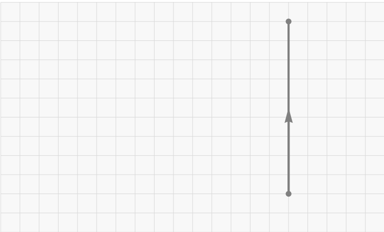
5. Compute the slope of the segment



6. Compute the slope of the segment

7. Compute the slope of the segment

8. Compute the slope of the segment



9. Compute the slope of the segment

10. Compute the slope of the line through points $(2, -3)$ and $(1, -1)$

11. Compute the slope of the line through points $(-3, 1)$ and $(-1, -2)$

12. Compute the slope of the line through points $(2, 3)$ and $(2, 3)$

13. Compute the slope of the line through points $(3, -2)$ and $(0, 1)$

14. Compute the slope of the line through points $(-3, -1)$ and $(-1, -1)$

15. Compute the slope of the line through points $(3, 1)$ and $(0, 2)$

0.3 Lines and Equations

Often it is very useful to understand and exploit the relationship between two quantities. We have seen this in previous sections. For example, we explored how the temperature in Celsius is related to the Fahrenheit temperature, and we saw how the time t is related to the height, h, of an object in projectile motion. Generally speaking, the study of the relationship between two quantities, x and y may bring fruitful insight. Moreover, a very good way to explore such relationship is with a graph where each value of x is plotted with its corresponding value for y. The result may be a collation of plotted ordered pairs which may be connected to show the pattern of the relationship. Such a diagram is often called *the graph of the equation.*

Example Graph the equation $y = 2x + 1$ by plotting points.

Solution:

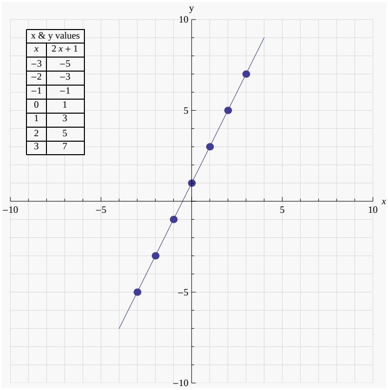
We begin by choosing various, somewhat random, values of x and for each one we find the corresponding y coordinate. If $x = 0$, then

$$y = 2(0) + 1 = 1$$

If $x = -1$, then

$$y = 2(-1) + 1 = -1$$

We continue this way, picking x values and fining respective y values.



We can organize them on a table, then plot and connect these points to complete the graph of the equation $y = 2x + 1$.

There are many ways one could write the equation of a line. In the previous example we wrote it as $y = 2x + 1$, but the same equation could also be written as $y - 2x = 1$ by simply adding a $-2x$ to each side. We could also write it as $y - 2x - 1 = 0$ and many other variations. As it turns out, writing it in the form

$$y = mx + b$$

for has particular advantages. This version of the equation is so famous it gets its own name, it is called the *Slope-Intercept Form* of the equation.

The key advantage to writing the equations in the $y = mx + b$ form is that such form reveals immediately all key information about the line. Namely, by simply looking at the equation in $y = mx + b$ form we can immediately see the place where it intercepts the y-axis *and* we can immediately see the slope of the line. Observe one more graph-by-plotting-points example, before we explain more.

Example Graph the equation $y = -x + 3$ by plotting points.

Solution:

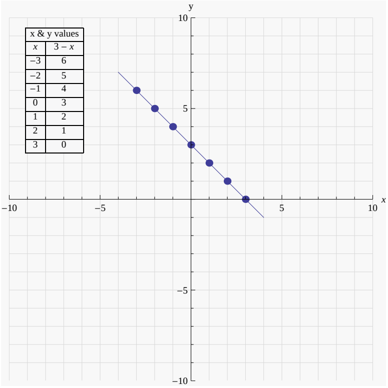
We begin by choosing various, somewhat random, values of x and for each one we find the corresponding y coordinate. If $x = 0$, then

$$y = -(0) + 3 = 3$$

If $x = 2$, then

$$y = -(2) + 3 = 1$$

We continue this way, picking x values and fining respective y values.



We can organize them on a table, then plot and connect these points to complete the graph of the equation $y = -x + 3$.

Now consider this last example, the graph of $y = -x + 3$ and note what happens when we set $x = 0$ The corresponding y value is

$$y = -0 + 3$$

This forces the point $(0, 3)$ to be part of the line. Moreover, this point, when $x = 0$, is exactly the point where the graph intercepts the y axis, namely it intercepts at $y = 3$. The same phenomena can be observed on the previous example. If we set $x = 0$ we find the y value where the graph intercepts the y axis

$$y = 2 \cdot 0 + 1$$

This is no accident, it will always be the case if we set $x = 0$ the mx vanishes to zero making b the y intercept *always* when the equation is written in

$$y = m \cdot 0 + b$$

form. In other words, when the equation is written in Slope-Intercept [SI] form, a quick glance is all that is need to see where it crosses the y axis. For example, we glance at

$$y = -3x + 4$$

and immediately we know the line crosses the y axis at $y = 4$ ..but wait, there is more! Now we turn our attention to the m in

$$y = mx + b$$

It turns out that the m is the slope of the line. To see why this is the case we can contemplate the question of what happens to the y value as we move from $x = 0$ to $x = 1$. We can plug these into the equation $y = mx + b$ to get the corresponding ys as

$$(0, b) \qquad (1, m + b)$$

implying that as the x's run 1 the ys rise from b to $m + b$ meaning the y's rise m leading to the inevitable conclusion that if we run 1, we rise m, thus the slope must be

$$slope = \frac{rise}{run} = \frac{m}{1}$$

Now we can revisit our first example, with a new deeper perspective. A simple glance at

$$y = 2x + 1$$

reveals the two key ingredients for the line; it crosses y at $y = 1$ and has slope $m = 2$.

We can do the same for our second example,

$$y = -x + 3$$

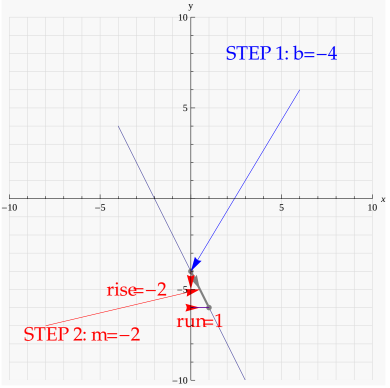
Such line must have slope $m = -1$ and y-intercept $b = 3$.

We can now use this perspective to graph lines, rather than by plotting several points, we simply plot the intercept and draw the slope from there.

Example Graph the equation $y = -2x + -4$ by using SI Form.

Solution:

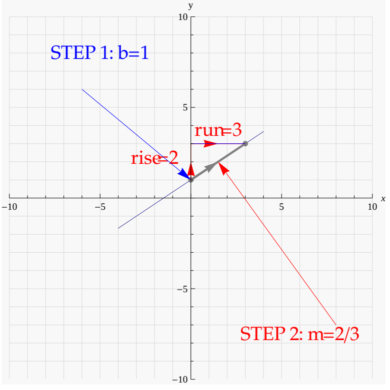
We need only two things 1)identify the intercept, draw the slope from there. We can see these from the equation, $y = \frac{-2}{1}x + -4$.



Example Graph the equation $y = \frac{2}{3}x + 1$ by using SI Form.

Solution:

We need only two things 1)identify the intercept, draw the slope from there. We can see these from the equation, $y = \frac{2}{3}x + 1$.



0.3.1 Exercises

1. Graph the equation of the line $y = \frac{5x}{2}$ by plotting points.

2. Graph the equation of the line $y = \frac{x}{4} + 1$ by plotting points.

3. Graph the equation of the line $y = \frac{3x}{2} - 2$ by plotting points.

4. Graph the equation of the line $y = \frac{3x}{5} - 2$ by plotting points.

5. Graph the equation of the line $y = x - 1$ by plotting points.

6. Graph the equation of the line $y = -\frac{x}{2} - 4$ by plotting points.

7. Graph the equation of the line $y = \frac{2x}{3} + 3$ by plotting points.

8. Write the equation $\frac{24x}{5} + 2\text{y} - 30 = 0$ in Slope-Intercept form, then use the SI form to graph it.

9. Write the equation $\frac{18x}{5} + 3\text{y} + 36 = 0$ in Slope-Intercept form, then use the SI form to graph it.

10. Write the equation $15x + 5\text{y} + 75 = 0$ in Slope-Intercept form, then use the SI form to graph it.

0.4 Point-Slope Form

Here is one way to summarize the previous section. If we know the y-intercept point $(0, b)$, and if we know the slope m, then we can describe the equation of the line. The equation in slope-intercept form is

$$y = mx + b$$

The rest of the section was devoted to studying such equation, its variations, and consequences.

From that point of view, this section is only a small tweak away. The tweak goes as follows. Suppose we still know the slope is m, but this time suppose we know some random point, (h, k), on the line. How then do we write the equation of such a line. In such case, any equation with real slope m, and going through the point (h, k) will correspond to the equation

$$y - k = m(x - h)$$

Such a form is called the *Point-Slope [PS]* form. As the name suggest, it works well because at a glance we can look at the equation and read the slope and a point on the line.

Example Find the slope and a point on the line described by the equation

$$y - 3 = \frac{1}{2}(x - 5)$$

Solution:

the slope is $\frac{1}{2}$ and the line goes through $(5, 3)$

Example Find the slope and a point on the line described by the equation

$$y - 2 = \frac{-2}{5}(x - -1)$$

Solution:

the slope is $\frac{-2}{5}$ and the line goes through $(2, -1)$

This works well backwards as well. If we know *one point* and the *slope* we can immediately write the PS form of the line.

Example Find the equation of a line with slope $m = -3$ and containing the point $(4, -1)$

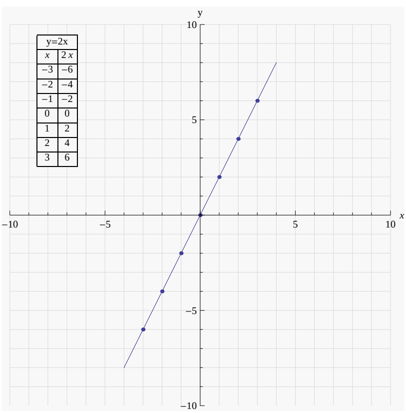
Solution:

the equation is

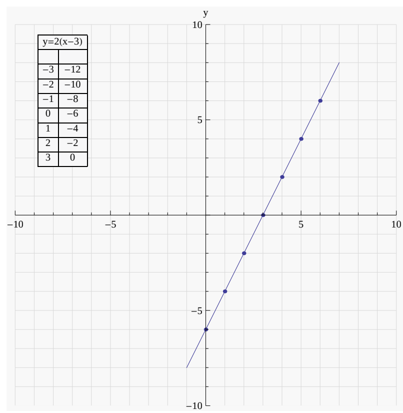
$$y - -1 = -3\,(x - 4)$$

The last item on our list is to shed some light into the mystery of this wonder-equation that makes it all too easy to from the line to the equation and backwards.

A powerful way to see this is to start with a simple graph such as the graph of $y = 2x$. We will then compare it to the graph of a slightly tweaked version of that equation. We replace "x" with "$x-3$", and see how the that affects the graph.

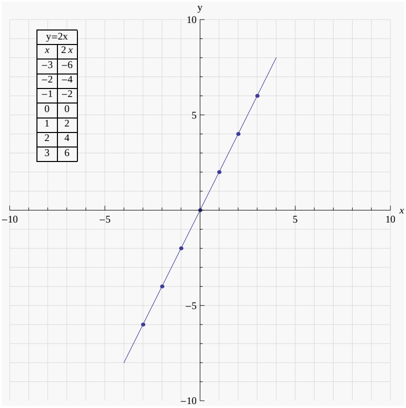


basic: $y = 2x$

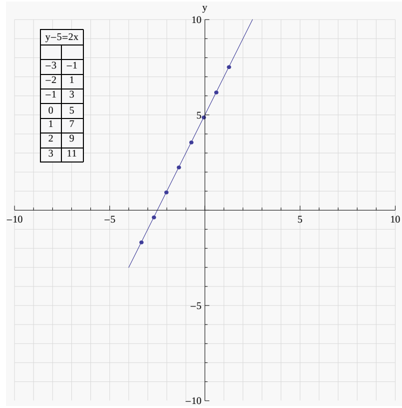


shifted right: $y = 2(x - 3)$

We see that the graphs are nearly identical, except that the one where "x" was replaced with "$x - 3$" go 'shifted' to the right 3 units. This is no accident. It is part of a much more general *shifting principle*. It world not just with lines, but with *any* equation with x's and y's . Moreover, it works as expected in the vertical direction. Meaning if we carry on the same experiment by replacing "y" with "$y - 5$" the graph would indeed be shifted up 2. Consider and compare the graphs below of equations $y = 2x$ as compared with the equation of $y - 5 = 2x$



basic: $y = 2x$

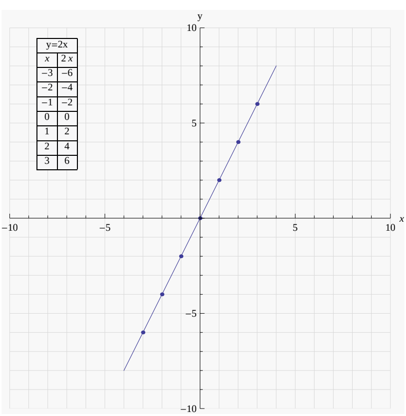


shifted up: $y - 5 = 2x$

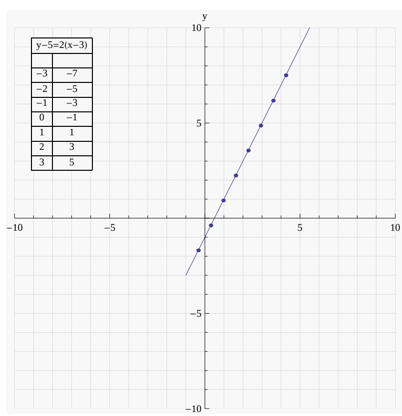
Putting these two ideas together, helps explain why the graph of $y - 5 = 2(x - 3)$ is a shifted version of the graph of $y = 2x$, shifted up 5 and 3 to the right, or more generally, this helps explain why the graph of

$$y - k = m(x - h)$$

is a shifted version of $y = mx$ as it is shifted k units vertically and h units horizontally.



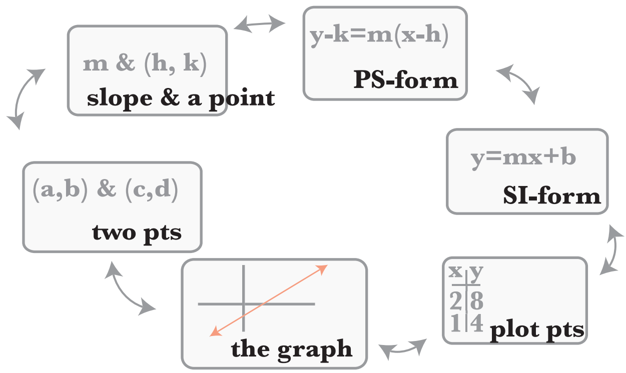
basic: $y = 2x$



shifted right 3 & up 5: $y - 5 = 2(x - 3)$

0.5 The Line Circle

By now, you should be comfortable thinking about slopes, y-intercepts, points-slope and graphs of linear equations in x and y. Here we review all these ideas and try to see the big picture. We will introduce no new concepts for now, instead, we simply emphasize fluency from one form of the equation to all the other forms.



With some exceptions, the above diagram gracefully summarizes all that we have done with lines. It should help us organize the ideas and how they are related. The main idea for this section is that if we know the information on any one of these boxes, we can derive all other forms of the equation. The diagram breaks down a bit if the slope is infinite or if the graph is not known with precision for example if it is not easy to identify any precise points on the line from the graph alone. In all other cases, knowing one item allows us to go all the way around.

Example From **PS-form** to **SI-form** Write the following in SI-form:

$$y - 3 = \frac{1}{2}(x - 6)$$

Solution:

SI-form requires that we solve for y [isolate y on one side] thus..

$$
\begin{aligned}
y - 3 &= \frac{1}{2}(x - 8) && \text{(given)} \\
y - 3 &= \frac{1}{2}x - \frac{1}{2}\cdot 8 && \text{(DL)} \\
y - 3 &= \frac{1}{2}x - 4 && \text{(BI)} \\
y &= \frac{1}{2}x - 1 && \text{(CLA, BI)}
\end{aligned}
$$

Example From **two points** to **slope and point** Find a point and the slope for a line through points $(2,5)$ and $(-1,10)$

Solution:

first we find the slope:

$$
\begin{aligned}
m &= \frac{rise}{run} && \text{(def of slope)} \\
&= \frac{10-5}{-1-2} && \text{(substitute)} \\
&= \frac{5}{-3} && \text{(BI)}
\end{aligned}
$$

now we are given two points on the line.. so to find one we need only choose one of these, either will do. We conclude with a point on the line, $(2,5)$ and the slope of the line $m = \frac{5}{-3}$

Example From **slope and point** to **PS-form** Find the PS-form for the line with $pt = (2,5)$ and $m = \frac{5}{-3}$

Solution:

we just plug in the items into the generic template

$$y - k = m(x - h)$$

thus the equation is

$$y - 5 = \frac{5}{-3}(x - 2)$$

0.5.1 Exercises

1. Suppose the points $\begin{pmatrix} 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} -1 & 4 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

2. Suppose the points $\begin{pmatrix} 2 & 6 \end{pmatrix}$ and $\begin{pmatrix} 4 & 7 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

3. Suppose the points $\begin{pmatrix} 4 & -3 \end{pmatrix}$ and $\begin{pmatrix} -8 & 0 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

4. Suppose the points $\begin{pmatrix} 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 2 & 2 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

5. suppose a line contains $\begin{pmatrix} -4 & -7 \end{pmatrix}$ and has slope $\frac{1}{2}$.

- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

6. suppose a line contains $\begin{pmatrix} \frac{2}{5} & -4 \end{pmatrix}$ and has slope $\frac{5}{2}$.

- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

7. Suppose the points $\begin{pmatrix} 1 & 2 \end{pmatrix}$ and $\begin{pmatrix} 6 & 2 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

8. Suppose the points $\begin{pmatrix} -2 & -4 \end{pmatrix}$ and $\begin{pmatrix} 6 & -4 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

9. Suppose the points $\begin{pmatrix} -4 & -1 \end{pmatrix}$ and $\begin{pmatrix} -4 & -1 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

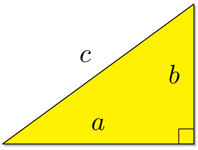
10. Suppose the points $\begin{pmatrix} 8 & 1 \end{pmatrix}$ and $\begin{pmatrix} 8 & 9 \end{pmatrix}$ are on a line.

- Find the slope and a point on the line.
- Find the PS-form of the line
- Find the SI-form of the line.
- make a table of points on the line [at least 5 points].
- Graph the line.

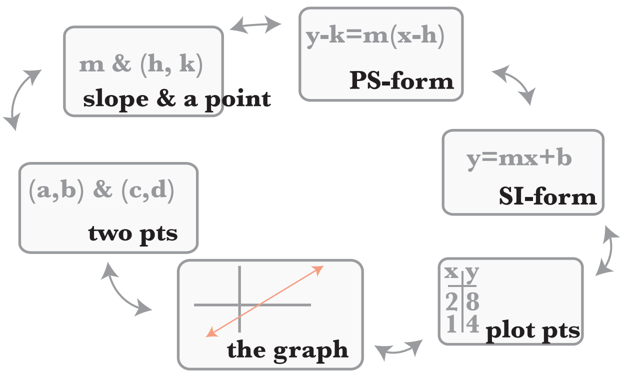
0.6 Chapter 6 Summary

[redacted]

$$a^2 + b^2 = c^2$$



[redacted]



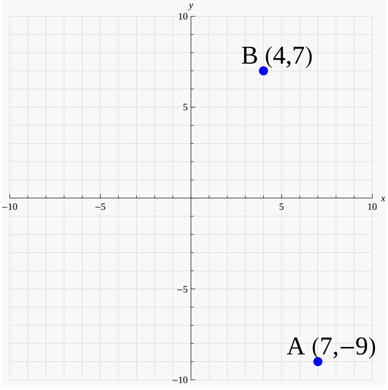
Some Answers

Section 0.1

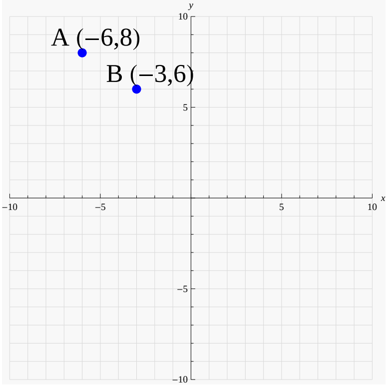
1. We use pythagoras!

$$\begin{aligned} (60)^2 &= (w)^2 + (7)^2 && \text{(given)} \\ w &= \pm\sqrt{3551} && \text{(CLA, SRP,bi)} \\ w &\approx \pm 59.59 && \text{(calc)} \end{aligned}$$

assume the w is positive, $w \approx 59.59$



2.

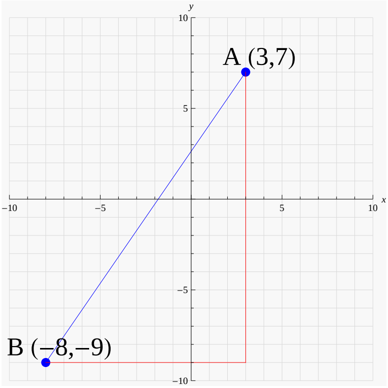


3.

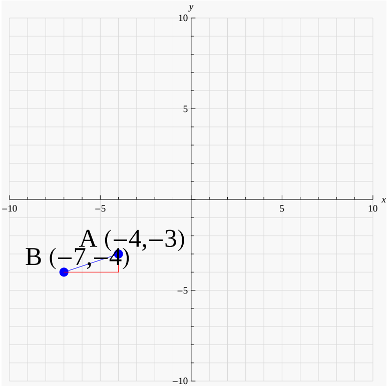
4. We use pythagoras!

$$\begin{aligned} (90)^2 &= (q)^2 + (75)^2 && \text{(given)} \\ q &= \pm\sqrt{2475} && \text{(CLA, SRP,bi)} \\ q &\approx \pm 49.75 && \text{(calc)} \end{aligned}$$

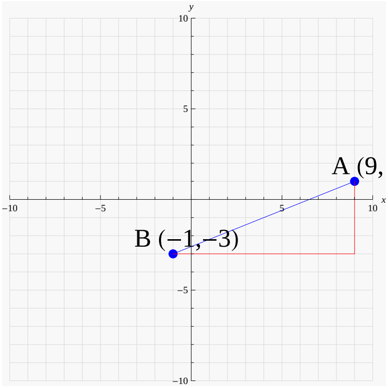
assume the q is positive, $q \approx 49.75$



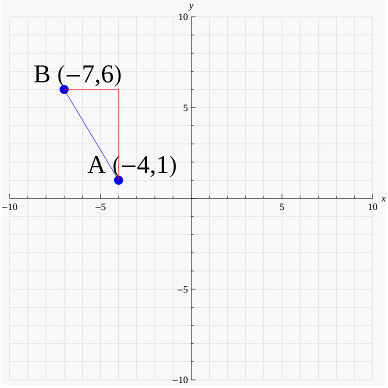
5. $d = \sqrt{377}$



6. $d = \sqrt{10}$



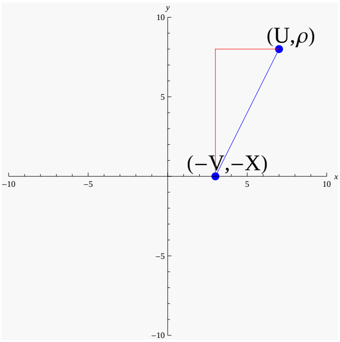
7. $d = 2\sqrt{29}$



8. $d = \sqrt{34}$

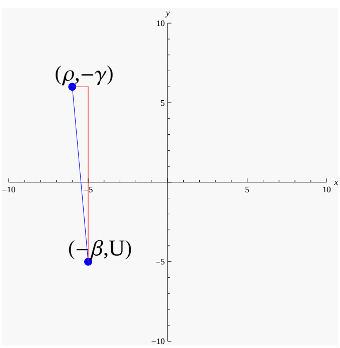
9.

$$d = \sqrt{(-U - V)^2 + (-\rho - X)^2}$$



10.

$$d = \sqrt{(-\beta - \rho)^2 + (\gamma + U)^2}$$



11. $x = 10$

14. cut the dotted lines, then use the noted dots, as pivot points to rotate the cut triangles, this should turn the $a^2 + b^2$ shape into a c^2 shape..

Section 0.2

1. slope is $m = 2$

2. slope is $m = \frac{1}{2}$

3. slope is $m = -\frac{4}{9}$

4. slope is $m = -\frac{3}{2}$

5. slope is $m = \frac{1}{13}$

6. slope is $m = -\frac{1}{6}$

7. slope is $m = 0$

8. slope is $m = \infty$

9. slope is $m = \infty$

10. slope is $m = -2$

11. slope is $m = -\frac{3}{2}$

12. slope is $m = \infty$

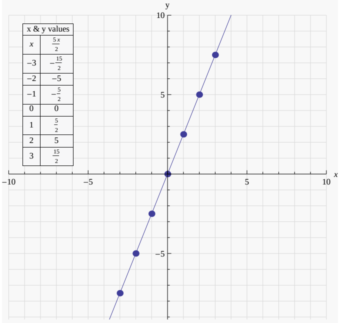
13. slope is $m = -1$

14. slope is $m = 0$

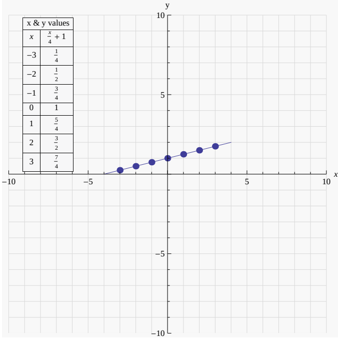
15. slope is $m = -\frac{1}{3}$

Section 0.3

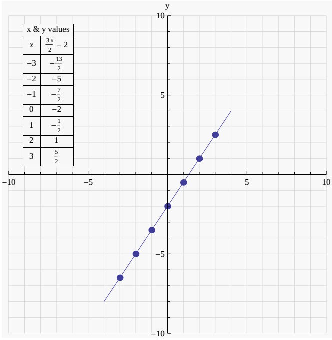
1.



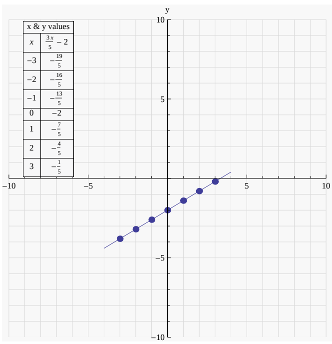
2.



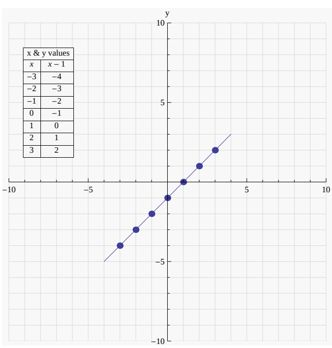
3.


x & y values
x | $\frac{3x}{2} - 2$
-3 | $-\frac{13}{2}$
-2 | -5
-1 | $-\frac{7}{2}$
0 | -2
1 | $-\frac{1}{2}$
2 | 1
3 | $\frac{5}{2}$
y
x

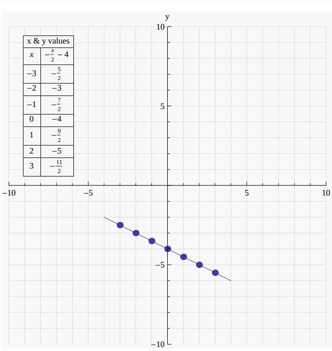
4.


x & y values
x | $\frac{3x}{5} - 2$
-3 | $-\frac{19}{5}$
-2 | $-\frac{16}{5}$
-1 | $-\frac{13}{5}$
0 | -2
1 | $-\frac{7}{5}$
2 | $-\frac{4}{5}$
3 | $-\frac{1}{5}$
y
x

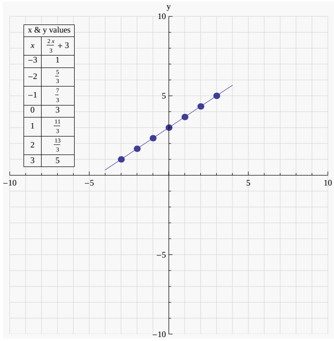
5.


x & y values
x | $x - 1$
-3 | -4
-2 | -3
-1 | -2
0 | -1
1 | 0
2 | 1
3 | 2
y
x

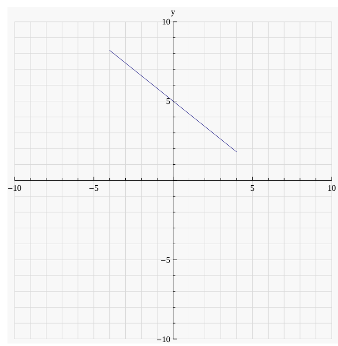
6.


x & y values
x | $-\frac{x}{2} - 4$
-3 | $-\frac{5}{2}$
-2 | -3
-1 | $-\frac{7}{2}$
0 | -4
1 | $-\frac{9}{2}$
2 | -5
3 | $-\frac{11}{2}$
y
x

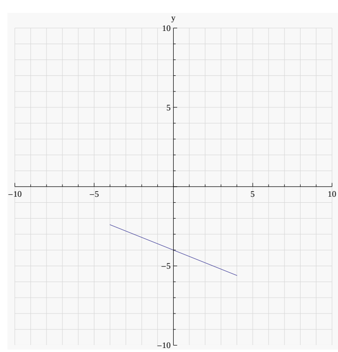
7.

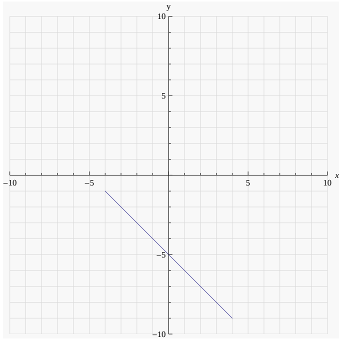


8. simplify to get $y = 5 - \frac{4x}{5}$ then...



9. simplify to get $y = -\frac{2x}{5} - 4$ then...

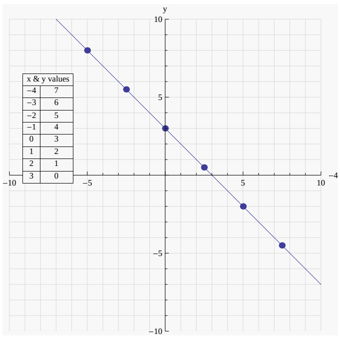




10. simplify to get $y = -x - 5$ then...

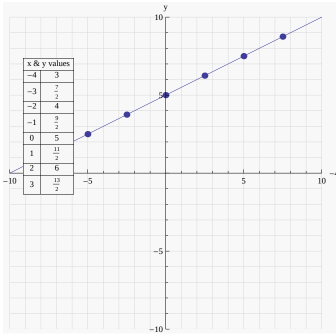
Section 0.5

1.

- one point is $\begin{pmatrix} 1 & 2 \end{pmatrix}$, and the slope is $m = -1$
- the PS-form is $y - 2 = -1\,(x - 1)$
- the SI-form is $y = 4 + 3$
- 

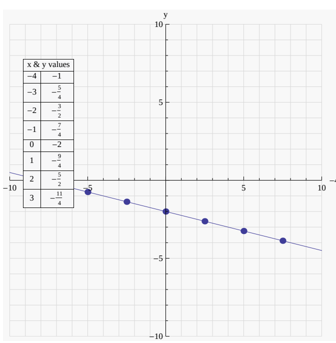
2.

- one point is $\begin{pmatrix} 2 & 6 \end{pmatrix}$, and the slope is $m = \frac{1}{2}$
- the PS-form is $y - 6 = \frac{1}{2}\,(x - 2)$
- the SI-form is $y = -2 + 5$

- 

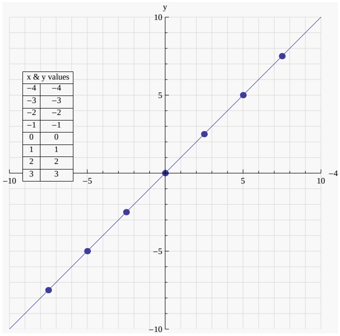
3.

- one point is $\begin{pmatrix} 4 & -3 \end{pmatrix}$, and the slope is $m = -\frac{1}{4}$
- the PS-form is $y - -3 = -\frac{1}{4}\left(x - 4\right)$
- the SI-form is $y = 1 - 2$

- 

4.

- one point is $\begin{pmatrix} 0 & 0 \end{pmatrix}$, and the slope is $m = 1$
- the PS-form is $y - 0 = 1\left(x - 0\right)$
- the SI-form is $y = -4 + 0$

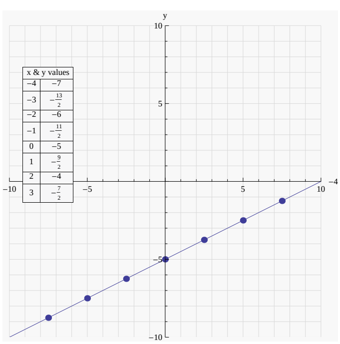
- 

5.

- one point is $\begin{pmatrix} -4 & -7 \end{pmatrix}$, and the slope is $m = \frac{1}{2}$

- the PS-form is $y - -7 = \frac{1}{2}\left(x - -4\right)$

- the SI-form is $y = -2 - 5$

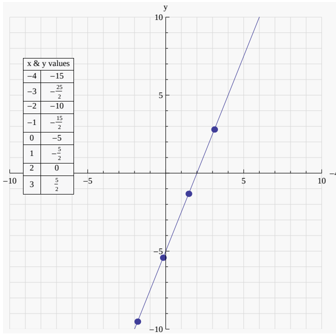
- 

6.

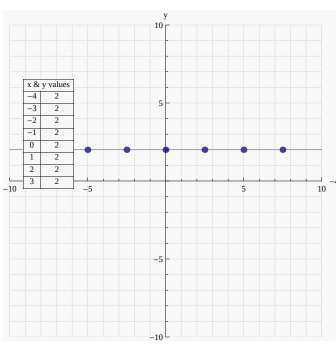
- one point is $\begin{pmatrix} \frac{2}{5} & -4 \end{pmatrix}$, and the slope is $m = \frac{5}{2}$

- the PS-form is $y - -4 = \frac{5}{2}\left(x - \frac{2}{5}\right)$

- the SI-form is $y = -10 - 5$

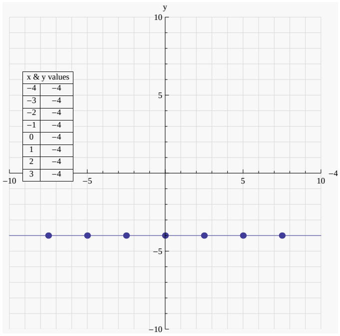
- 

7.

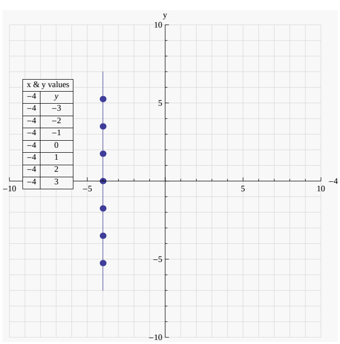
- one point is $\begin{pmatrix} 1 & 2 \end{pmatrix}$, and the slope is $m = 0$
- the PS-form is $y - 2 = 0\,(x - 1)$
- the SI-form is $y = 2$
- 

8.

- one point is $\begin{pmatrix} -2 & -4 \end{pmatrix}$, and the slope is $m = 0$
- the PS-form is $y - -4 = 0\,(x - -2)$
- the SI-form is $y = -4$

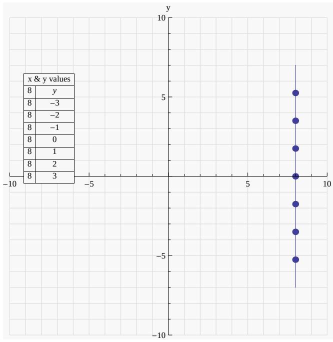
- 

9.

- one point is $\begin{pmatrix} -4 & -1 \end{pmatrix}$, and the slope is $m = \infty$
- the equation has no PS-form , only $x = -4$
- the equation has no SI-form, only $x = -4$
- 

10.

- one point is $\begin{pmatrix} 8 & 1 \end{pmatrix}$, and the slope is $m = \infty$
- the equation has no PS-form , only $x = 8$
- the equation has no SI-form, only $x = 8$

- 
y
x & y values
8 | y
8 | −3
8 | −2
8 | −1
8 | 0
8 | 1
8 | 2
8 | 3
10
5
−5
−10
−10
−5
5
10
8

Bibliography

[1] Wikipedia Euclid's Elements Retrieved August 10, 2011

[2] Wikipedia Georg Cantor Retrieved August 2011

[3] Wikipedia Cantor's Diagonal Argument Retrieved August 2011

[4] Elisha S Loomis The Pythagorean Proposition National Council of Teachers of Mathematics (1972)

Index