REAL NUMBERS

Roughly speaking, the real numbers are all numbers which can be approximated infinitely-closely by rational numbers. The real numbers are often represented by a number line. Every point on the number line can be approximated by a fraction, and conversely, thus the number line is, roughly speaking, a good portrait of the real numbers.

\[
\begin{array}{cccc}
& -2 & -1 & 0 & 1 & 2 \\
\end{array}
\]

SQUARE ROOTS

For a non-negative real number \( a \), we define the square root of \( a \) to be the non-negative solution of
\[
x^2 = a
\]
We denote such number by \( \sqrt{a} \) (read 'square root of a' or 'radical a'). We accept, as definition, that
\[
(\sqrt{a})^2 = a \quad [\text{def SQR}]
\]

CUBIC ROOTS

For a real number \( a \), we define the cubic root of \( a \) to be the real solution of
\[
x^3 = a
\]
We denote such number by \( \sqrt[3]{a} \) (read 'cubic root of a'). We accept, as definition, that
\[
(\sqrt[3]{a})^3 = a \quad [\text{def CR}]
\]

MONOMIAL

If \( A \) is a real number and \( n \) is a whole number then
\[
Ax^n
\]
is called a monomial, \( A \) is called the coefficient, \( x \) is called the variable, and \( n \) is called the degree. Monomials of degree 0 are often call constants, degree 1 linear, degree 2 quadratic, and degree 3 are often called cubic.

POLYNOMIAL

A polynomial is a monomial or the sum or difference of several monomials, often called terms. When simplified, the degree of a polynomial is the highest of all degrees of its terms.

Famous Polynomials

1. \( a^2 - b^2 = (a - b)(a + b) \) Difference of two Squares [DS]
2. \( a^3 - b^3 = (a - b)(a^2 + ab + b^2) \) Difference of two Cubes [DC]
3. \( a^3 + b^3 = (a + b)(a^2 - ab + b^2) \) Sum of two Cubes [SC]
4. \( a^2 + b^2 = \) coming soon..
Pascal Polynomials

1. \((a + b)^2 = a^2 + 2ab + b^2\)  
   \[[PP\#2]\]

2. \((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\)  
   \[[PP\#3]\]

3. \((a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\)  
   \[[PP\#4]\]

4. \((a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5\)  
   \[[PP\#5]\]

Geometric Polynomials

1. \(x^2 - 1 = (x - 1)(x + 1)\)  
   \[[GS\#2]\]

2. \(x^3 - 1 = (x - 1)(x^2 + x + 1)\)  
   \[[GS\#3]\]

3. \(x^4 - 1 = (x - 1)(x^3 + x^2 + x + 1)\)  
   \[[GS\#4]\]

4. \(x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)\)  
   \[[GS\#5]\]

SET of POLYNOMIAL

The set of all polynomials with real number coefficients and variable \(x\) is often denoted by the symbol \(\mathbb{R}[x]\).

restricted SET of POLYNOMIAL

If the coefficients are restricted to include only integers (i.e. \(\mathbb{Z}\)), the corresponding set of polynomials with variable \(x\) is often denoted by the symbol \(\mathbb{Z}[x]\).

Multiplying POLYNOMIALs [KG]

\[
(-2x + 1)(5x^2 + 4x - 1) = 5x^2 + 4x + 1 \times (-2x + 1) \\
\]

\[
= 5x^2 + 4x + 1 \\
- 10x^3 - 8x^2 + 2x \\
\]

\[
= -10x^3 - 3x^2 + 6x + 1 
\]

Dividing POLYNOMIALs [LD]

Example:

\[
2x - 1 \) \frac{5x^2 + 16x + 3}{10x^3 + 27x^2 - 10x} \]

\[
- \frac{32x^2 - 10x}{6x} \\
- \frac{-6x + 3}{3} \\
\]

\[
\frac{x^2 + x + 1}{x - 1} \) \frac{x^3 - x^2}{-x^3 + x^2} \frac{-x^2 + x}{x - 1} \frac{-x + 1}{0} \\
\]

FOIL

\[
(a + b)(c + d) = ac + ad + bc + bd 
\]

[FOIL]