1. Convert 300°

2. Convert 30°

3. How many radii fit on the circumference of half a circle?

4. How many radii fit on an entire circle

5. If the radius of a circle is 5 inches long, how long is the circumference of half of the circle?

6. Convert 1

7. Convert 3°

8. Convert 1°

9. Convert $\frac{-5\pi}{6}$

10. What is one radian?

11. Convert 170°

12. If the radius of a circle is 7 inches long, how long is the circumference of the entire circle.

13. Convert 4

14. Convert 3

15. Convert $\frac{5\pi}{12}$

16. Convert 50°

17. Determine if:

$$\pi \approx 3.14$$

A. TRUE B. FALSE

18. Convert 60°

19. Convert 225°

20. If the radius of a circle is 5 inches long, how long is the circumference of the circle?

21. Determine if:

$$\pi = 3.14$$

A. TRUE B. FALSE

22. Convert $1\,rad$

23. If the radius of a circle is *blah* inches long, how long is the circumference of the entire circle.

24. Convert $\frac{11\pi}{12}$

25. Convert 390°

26. Convert 2

27. Convert $\frac{\pi}{12}$

28. If the radius of a circle is r inches long, how long is the circumference of the entire circle.

29. What is one degree?

30. Convert 10°

The Main Idea

In this section we discuss important ideas and vocabulary associated with angles. We will discuss the two most common units of measurement for angles, degrees and radians. We will measure famous angles using both units, learn how to convert measurements from radians to degrees and vise-versa. We also introduce timeless ideas such as the concept of π.

Units of Measurement

Let us be clear, one central idea in trig is to be able to measure distances far away, such as the hight of a building, for measuring angles close by, such as the angle one makes by considering the lines of sight from the top to the building and to the bottom of the building.

The underlying reason for the interest in this is of course because small angles make small opposite sides while larger angles make larger opposite sides. Dominating this relationship implies being able to take measurements of things from very far away, and this is precisely the essence of trigonometry.

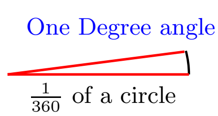
This is precisely what we will do in this course. Moreover, to carry out this amazing feat we must become good at measuring angles. *"Little angle"*, *"medium"* or *"really big angle"* are descriptive terms but we can do much better. Indeed, we do much better by developing "units of measurement" for angles.

We will adopt two such units, *degrees* and *radians*. We will define and explain exactly what each one of these units represents. We will then consider famous angles and their respective famous measurements both in radians and degrees. We will then conclude with take apart and digest the timeless concept of π (pronounced 'pi').

Degrees

Imagine taking a circle, then slicing it up in half, then in quarters, etc... until there are 360 slices of equal size. *One Degree* is by definition the measurement of the angle for each of these slices. It may be interesting to note that there 360 is in some ways an arbitrary number which well cold have been 100 or 1000. For whatever, reason 360 was the number of slices needed to make each one "one degree". These "degrees" have remained popular since inception and are now part of our everyday language (ie we make an 180 turn.. etc.). Let us consider some famous angles measured in degrees.

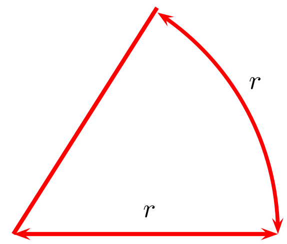
Degrees are units of measurement for angles. *One degree* is defined to be the measurement of the angle created by slicing up a circle into 360 equal slices.



Radians

Radians are in some ways more natural than degrees. To understand what a radian is, we take a look at how one such *radian* is 'cooked'. To do so, we begin with a circle with radius r, where r is some positive real number. We then imagine measuring this same distance r, along a portion of the circle, marking the endpoints. From each of these endpoints, we draw a segment to the center of the circle, constructing a very important slice of the circle. Such slice should look something like this.

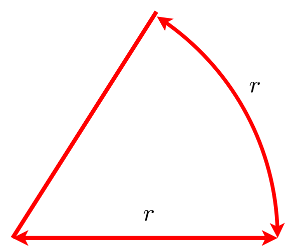
An important slice of a circle, carefully constructed



The angle created by this slice is precisely what we will define as the measurement of *one radian*. Said differently, *one radian* is defined to be the measurement of the angle created by the stretching along a circle the length of one *radius*.

Radians are units of measurement for angles. *One Radian* is defined to be the measurement of the angle created by the distance of one radius along part of the circumference of a circle.

One Radian



This discussion turns into some timeless ideas as we ask some innocent little questions. For example, we noted before the number of *"one-degree slices"* that fit in a circle is 360, by definition. Thus a half-circle contains exactly 180 one-degree slices, or said differently, the half-circle angle measures 180°. We now ask the same question for radians: *how many one-radian slices fit in the half-circle?*. This undecorated question leads to big, timeless, and very decorated ideas. As it turns out we can sketch these slices and see that roughly 3 such slices fit in the half-circle. But if we are extremely careful and accurate in our drawings we realize 3 such slices fall short of the half-circle, thus we can conclude that a little more than 3 radians fit in the half-circle. We can take portions of a radius and fit them in to conclude that 3.1 radians fit in a half-circle, but actually there is still room left for more. Depending on our instruments we may be able to realize that 3.14 radians fit in a half-circle, but there is a little more room left. The question of exactly how many radians fit in a half-circle consumed bright minds and ancient civilizations for centuries. Some civilizations never got to a satisfactory answer to this while others answered it incorrectly. Using some very deep ideas beyond the scope of this course, we now know the number starts off as

$$\pi = 3.14159265358979323846264338327950288\ldots$$

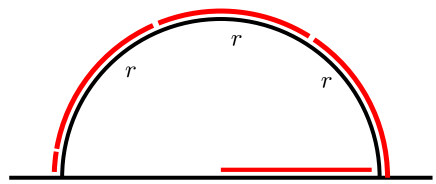
The trouble is that is does not end there, and to stop there would be inaccurate. In fact, it goes on for millions and millions and more digits. The contemporarily prevailing resolution has been to recognize that this number can not be named by its digits-representation due to the fact that it has too many digits without much of a pattern. Instead, we name this number and refer to it by the symbol "π".

Definition of π

Thus, the number that can not be named by digits, π, was born. It was born to be *the exact number radians that fit in a half-circle.* We emphasize:

π is defined to be *the exact number of times the radius of a circle fits in half the circumference of the circle.*

OWN π



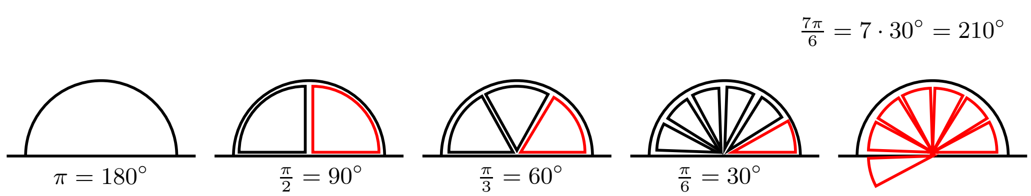
π = *exact* number of r's that fit in a half-circle

$\pi \approx 3.14$

Famous Angles: Degrees & Radians

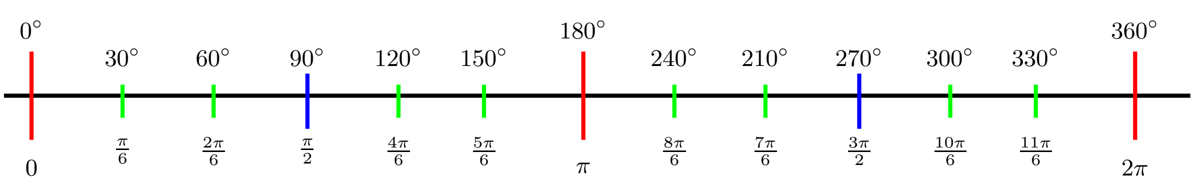
The above definitions of a degree, a radian, and π lead to some famous consequences. For example, if by definition, there are 360° in a circle, then there are 180° in a half-circle, and 90° in a quarter circle. By similar logic, there are π radians on a half-circle, 2π in a complete circle, and $\frac{\pi}{2}$ radians in a quarter of a circle. This leads to famous conversions, π radians can be converted to 180° just as $2\pi rad = 360°$, or $\frac{\pi}{2} = 90°$. It should be noted that when an angle measurement is given with no specified units, the default units are by default and tradition radians.

Here is a visual representation of some famous angles converted expressed in both degrees and in radians.



Famous: Degrees & Radians

Here is another way to visualize the conversion of famous degrees and their respective famous counterparts measured in radians.



Now we convert a non-famous angle. The key idea to convert units from/to degrees to/from radians is the observation, by definition of degrees and radians [DDR]:

$\pi\,rad = 180\,deg$ thus.. $1 = \frac{\pi\,rad}{180\,deg}$ and $1 = \frac{180\,deg}{\pi\,rad}$

Converting can be achieved by multiplying by the appropriate versions of '1'. To convert from degrees to radians one would multiply by '1' written as $1 = \frac{\pi\,rad}{180\,deg}$. On the other hand, to convert from radians to degrees one would multiply by '1' written as $1 = \frac{180\,deg}{\pi\,rad}$

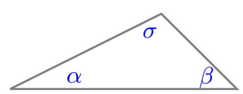
example: Convert $75\,deg$

$$
\begin{aligned}
75\,deg &= 75\,deg \cdot 1 && \text{(MiD)} \\
&= 75\,deg \cdot \frac{\pi\,rad}{180\,deg} && \text{(DDR)} \\
&= 75 \cdot \frac{\pi\,rad}{180} && \text{(Bi, note 'deg' no more)} \\
&\approx 1.309\,rad && \text{(Bi)}
\end{aligned}
$$

Interior Angles Theorem

The following theorem will be used frequently in our course. It states that in each of the triangles we will study, the sum of the interior angles is exactly 180°. It should be noted that this theorem works well for triangles on *flat* surfaces and may not hold true for non-flat surfaces such as the surface of a sphere. Moreover, the student is encouraged to explain or attempt to explain why it is so.

For any triangle in Euclidian Space, the sum of the interior angles is 180°
i.e. for α, β and σ below



then

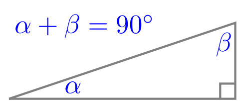
$$\alpha + \beta + \sigma = 180^\circ$$

e con-

clude this section with a couple words to add to our vocabulary.

If the measurement of two angles adds to 90°, these angles are called *complimentary*

Note: on a right triangle, the two non-right angles are complimentary. i.e. α and β below



If the measurement of two angles adds to 180°, these angles are called *supplementary*.

Note: on a right triangle, the two non-right angles are complimentary. i.e. α and β below