1. If a track has the shape of a circle with radius of 5 miles. How many miles is the entire track?

2. If a track has the shape of a circle with radius of 5 miles. How many miles is half of the track?

3. If a track has the shape of a circle with radius of 5 miles. How many miles is one third of the track?

4. If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of $65°$ of the track?

5. If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of $253°$ of the track?

6. (super famous!!) If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of $x°$ of the track?

7. If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of 1 radian of the track?

8. If a large pizza has a radius of 10 inches. How many 1-inch square can one make from a 39 deg portion of one pizza?

9. If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of 1.5 radians of the track?

10. If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of 10 radians of the track?

11. (super famous!!) If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of θ radians of the track?

12. If a large pizza has a radius of 10 inches. How many 1-inch square can one make from one pizza?

13. If a large pizza has a radius of 10 inches. How many 1-inch square can one make from half of one pizza?

14. If a large pizza has a radius of 10 inches. How many 1-inch square can one make from $\frac{4}{5}$ of one pizza?

15. If a track has the shape of a circle with radius of 5 miles. How many miles is a portion of 2 radians of the track?

16. If a large pizza has a radius of 10 inches. How many 1-inch square can one make from a 100 deg portion of one pizza?

17. (super famous!!) If a large pizza has a radius of r inches. How many 1-inch square can one make from a x deg portion of one pizza?

18. If a large pizza has a radius of 10 inches. How many 1-inch square can one make from a 2 radian portion of one pizza?

19. If a large pizza has a radius of 10 inches. How many 1-inch square can one make from a 3 radian portion of one pizza?

20. (super famous) If a large pizza has a radius of r inches. How many 1-inch square can one make from a θ radian portion of one pizza?

The Main Idea

In this section we consider ideas that will help us calculate the area of a circle, and/or the area of a part of a circle. We will also learn how to calculate the circumference of a circle or a portion of a circle. We start with a layman approach, consider a circle of radius 4:

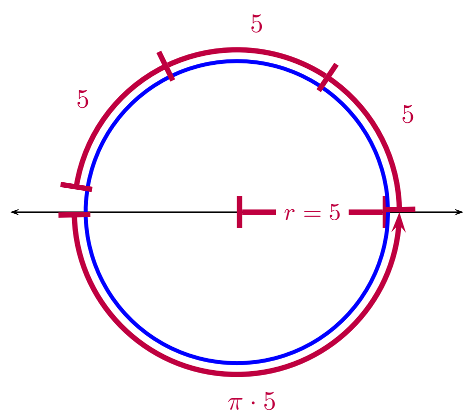
Circumference of a Circle

We can begin with an actual example. Suppose a circle is given with radius 4 inches. We will make use of the fundamental definition of π. Recall, π is the exact number of radians in half a circle. That is, π is the exact number of times the radius distance, $5in$, can fit along the circumference in half the circle.

From this we gather that the circumference of the circle, by the very definition of π is the radius distance multiplied π times for half the circle, thus 2π for the entire circle. This leads to the very famous and important formula for the circumference of a circle namely

$$C = 2\pi r$$

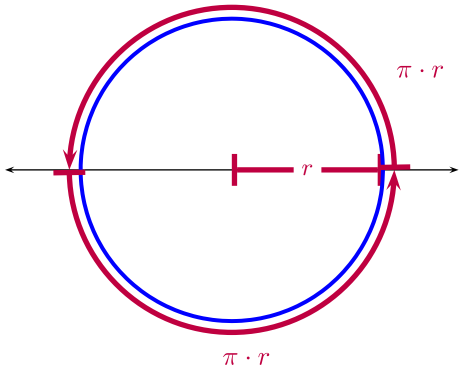
half circumference is ***approximately*** three 5-inch radii



half circumference is ***exaclty*** $\pi \cdot 5$-inch radii

Since each of the half-circles has length $\pi \cdot 5$, the total circumference is given by $\pi \cdot 5 + \pi \cdot 5$ for a total of $2 \cdot \pi \cdot 5$. This general idea is a well known formula for the circumference of a circle and its is summarized below.

half circumference is ***exaclty*** $\pi \cdot r$ units radii



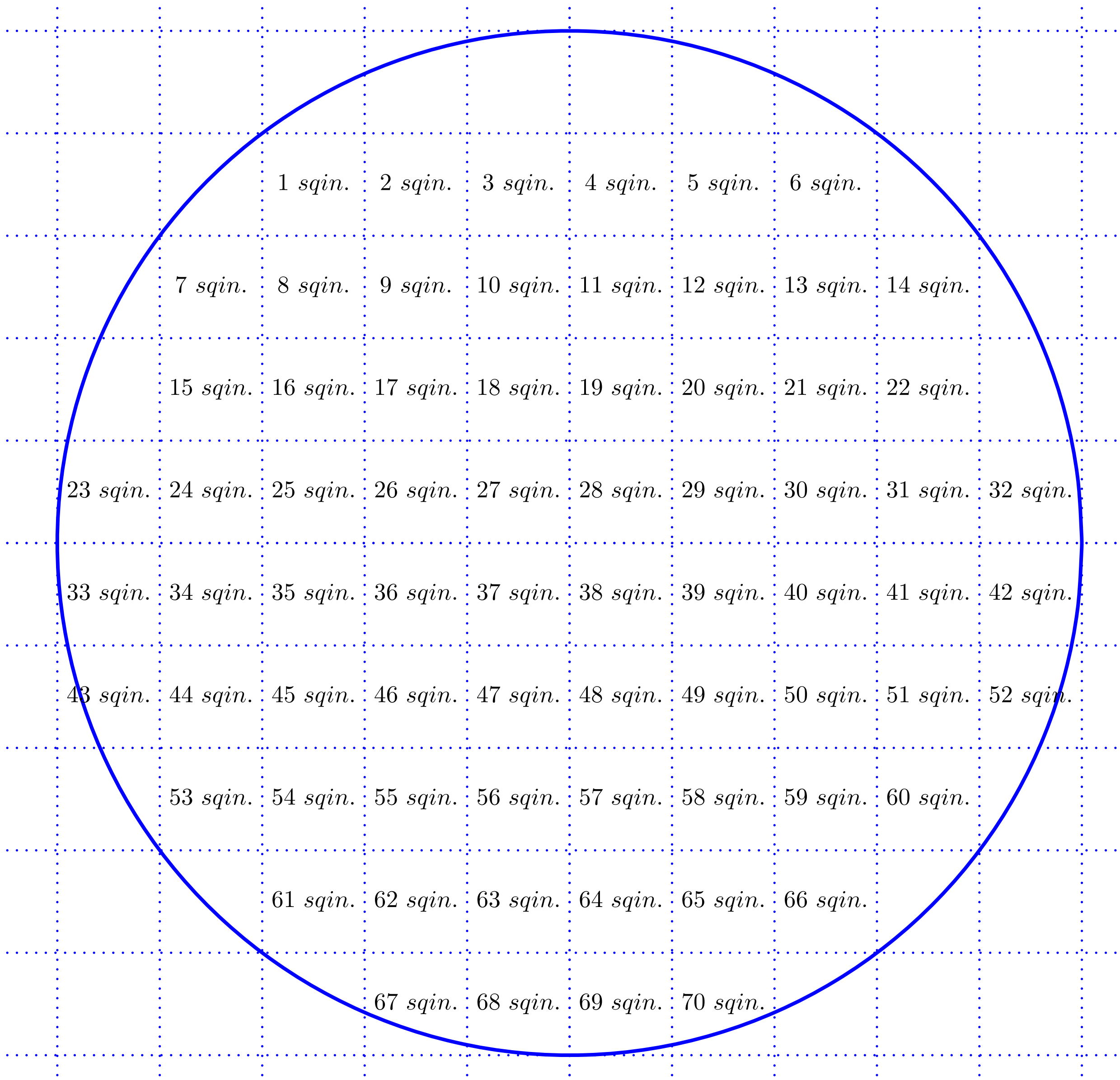
half circumference is ***exaclty*** $\pi \cdot r$ units radii

$$C = 2\pi r$$

Approximate Area of a Circle: layman approach

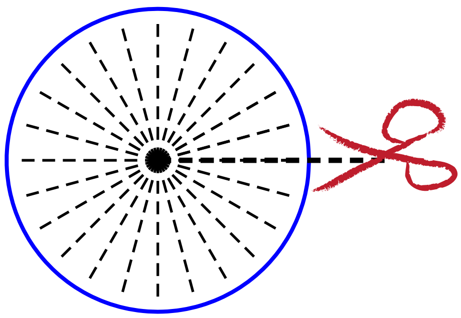
Now, we turn our attention to the area of a circle. One elementary approach would be to simply start counting how many 'little boxes' are contained in the circle. This should be easy to do and easy to understand, and also easy to approximate. Let us consider, for example, a circle with radius 5 in. or said differently, imagine a pizza with diameter of 10 inches, and ask how many 1-inch squares can be cut out from the pizza? Suppose we go on a do the most primitive thing and we simply begin to *simply count how many square inches* fit inside.

Counting The Approximate Area of a Circle: *how many square inches in a 10in pizza[radius 5 in]?*



At this point two things should be clear; one, that there are probably more than 70 sq. inches in such pizza and two, that the exact amount is a bit elusive under this technique since there are some portions of squares that were counted and should not have been and there were some portions not counted that should have been.... None the less, this is a common sense approach, one worth noting and a perfect appetizer for more precise, graceful, and inspired idea.

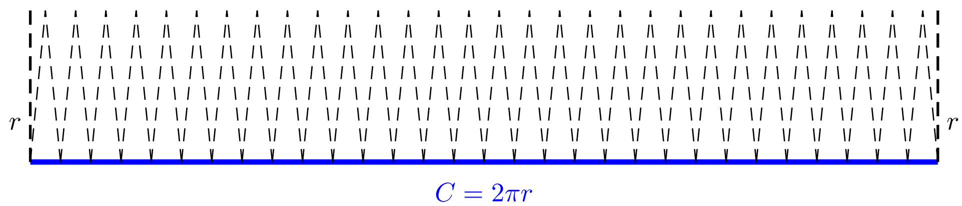
Area of a Circle: Step One Cut



THE Approach

The previous layman approach was just to warm up to the problem at hand. Now we take a bold an creative leap. This is THE approach we will follow to eventually conclude with the very famous area formula for a circle. We begin to cut up the circle into many, many pieces. The first cut, from outside directly to the center, the rest from the center towards almost the outside. As such:

Area of a Circle: Step Two Open & Strech



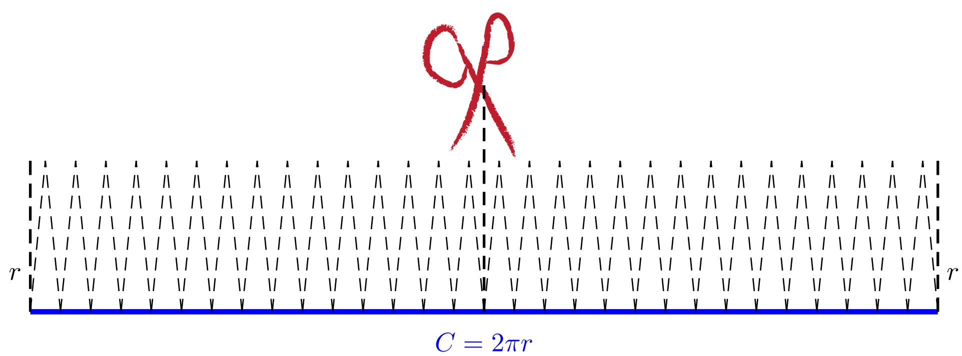
NOTE

The blue line still reprecents the circumference,

$$C = 2\pi r$$

while the hight of the sashed lines is still the radius, r

Area of a Circle: Step Three Cut

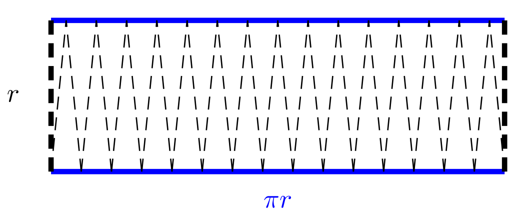


JUST CUT

We cut vertically, right through midpoint of the the Circumference

$$C = 2\pi r$$

Area of a Circle: Step Four Fit & Re-arrange



Yippee-Kay-Yee

Now, the area of the circle, which we have cut into a rectangle, is base times hight: Thus

$$A = \pi r^2$$

Some of the Consequences

The two formulas presented here, for the area and circumference of a circle, are timeless, and they have timeless consequences. In particular, these formulas lead to other well known formulas to calculate portions of the area of a circle and/or portions of the circumference of a circle. These formulas come in two flavors one for degrees and one for radians. Rather than simply present such formulas here, we invite the student to own these formulas by working on the following exercises. .