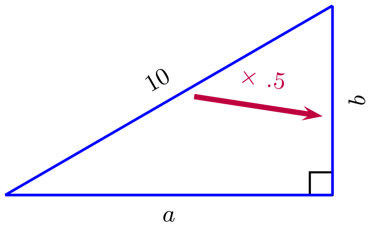
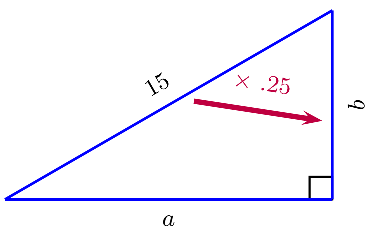
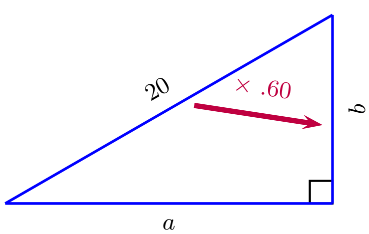
1. Solve the sides of the following triangle.



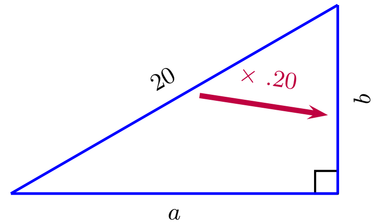
2. Solve the sides of the following triangle.



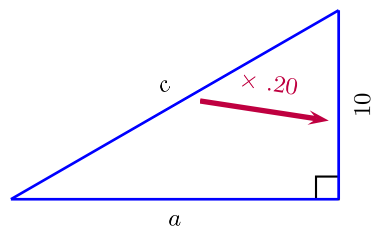
3. Solve the sides of the following triangle.



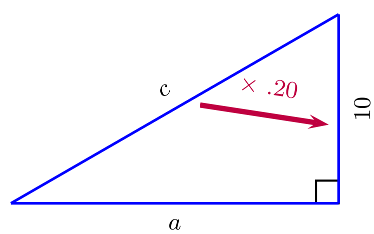
4. Solve the sides of the following triangle.



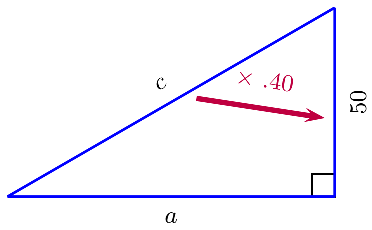
5. Solve the sides of the following triangle.



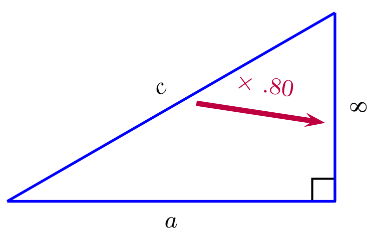
6. Solve the sides of the following triangle.



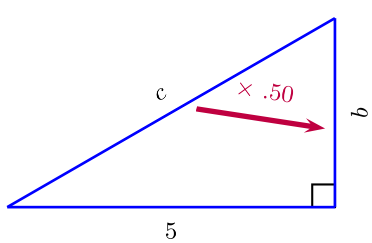
7. Solve the sides of the following triangle.



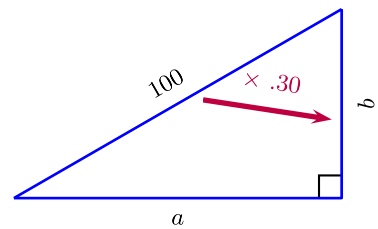
8. Solve the sides of the following triangle.



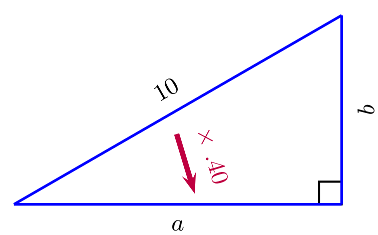
9. Solve the sides of the following triangle.



10. Solve the sides of the following triangle.



11. Solve the sides of the following triangle.

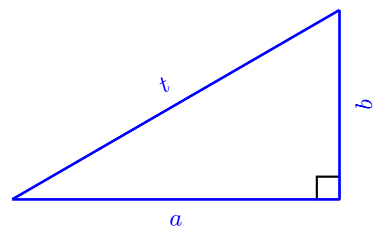


The Secret to Solving triangles

Well, this is just the first sentence and it is already time to come clean. The title on this section is a purposefully deceiving. There really are many nice secrets that can help us solve triangles. There is not one "*the* secret", yet this one is so important that calling attention to it by *any* means possible is warranted, honesty begone!

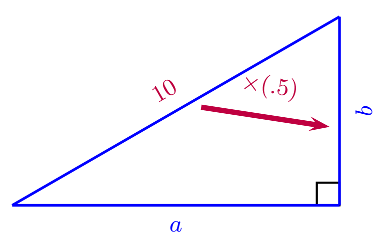
Consider one of the most important promises of trigonometry is the ability to solve triangles and thereby be able to measure distances from far away. This can roughly be translated to solving all sides of a triangles [even the sides that may be far away and not ready 'measurable']. Thus, the importance of *the secret sauce*. More than a sauce its an idea.

Suppose we have at hand a Euclidean right triangle, a standard one with two sides, one right angle, and two complimentary angles, two legs and a hypothenuse, etc, etc.



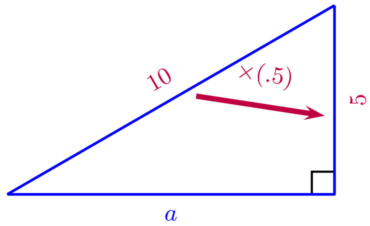
Now, *the secret sauce* idea is the idea that if we know, or if we could figure out just *one side & one ratio* of the sides of the triangle, then, it would be all over and triumph would be ours. That is, if we knew just *one side & one ratio* we could take it from there and figure out the rest of the triangle all of the rest, meaning the other two sides and eventually the angles as well.

One Side & One Ratio



That is it! given the above information, namely, we have a side and a ration on a right triangle, we can figure out the rest of the triangle. This is indeed the secret sauce.

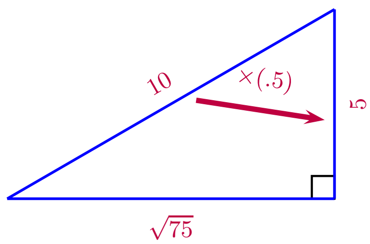
Observe. First let us understand the ration part. The arrow is intended to illustrate that the ration between the hypothenuse and side b is 50%. In other words, to go from the hypothenuse to side we we need only *multiply by .5*. Yet another way to put it is: to go from side b to the hypothenuse we multiply by 2. Having said that, since the hypothenuse is given to be 10, we multiply by .5 to obtain $b = 5$ Now we have



Which gives us two side of a right triangle. Now pythagoras takes over. Pythagoras would lead us to:

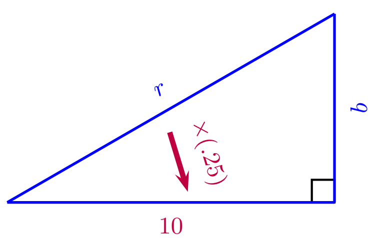
$$5^2 + a^2 = 10^2$$

which would eventually lead us to $a^2 = 75$ and $a = \sqrt{75}$, thus we have solved, from just one side and one ration ALL sides of the right triangle:

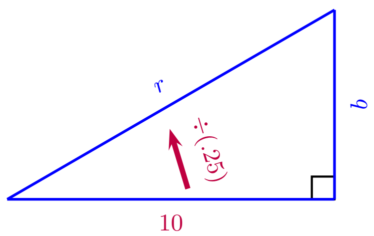


One Side & One Ratio

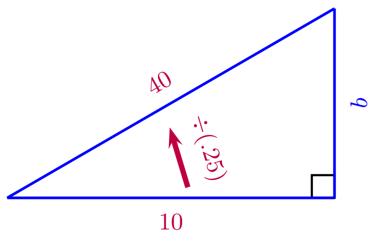
We now try our luck with another example. Suppose again we have a standard right triangle and one side and one ration are known.



Let us do a bit of interpreting again. The diagram implies that to go from r we would *multiply by* .25 to get to 10. This means that to backwards from the side labeled 10 to r we can *divide by* .25 Thus the above diagram is equivalent to this diagram:



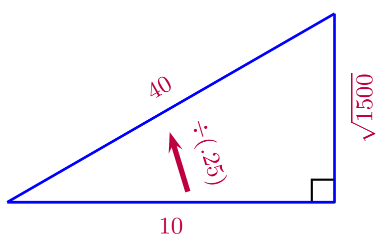
Thus we divide 10 by .25 to obtain $r = 40$ and now we have two of the sides.



We now invoke the powers of pythagoras to deliver for us the third side:

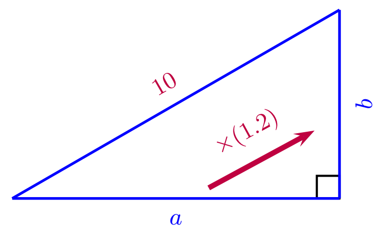
$$10^2 + b^2 = 40^2$$

Leading to $b = \sqrt{1500}$ and now we have the complete picture:

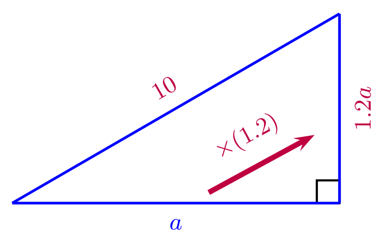


One Side & One Ratio: TAKE 3

This time we consider again, a right triangle where one of the sides and one of the rations are given but this times suppose the give ratio does not involve the given side.



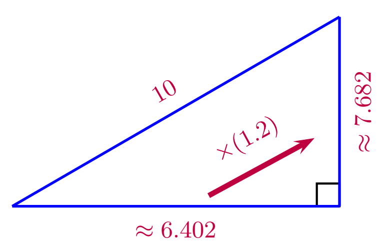
To solve this triangle, we note the given ratio implies that b is 1.20% of a, or that a times 1.20 is equal to b. This helps us know something about b, namely that $b = 1.2a$ thus we have,



We then apply pythagoras:

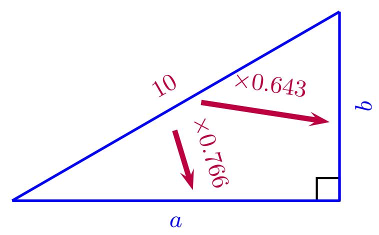
$$
\begin{aligned}
a^2 + (1.2a)^2 &= 10^2 && \text{(pyth)} \\
a^2 + 1.44a^2 &= 10^2 && \text{(algebra)} \\
2.44a^2 &= 100 && \text{(algebra)} \\
a^2 &= \frac{100}{2.44} && \text{(algebra)} \\
a &= \pm\sqrt{\frac{100}{2.44}} && \text{(algebra)} \\
a &\approx 6.402 && \text{(assume positive)}
\end{aligned}
$$

Now, we know that the ratio b to a is 1.20% thus $b = 1.2a \approx 1.2(6.402) = 7.6824$ Therefore, we have solve the triangle as promised, from *one side and one ratio*

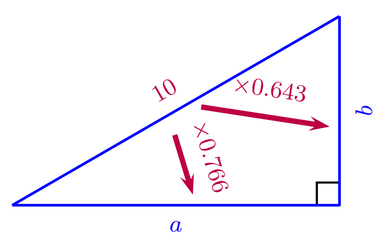


One Side & Two Ratios: Even Better!

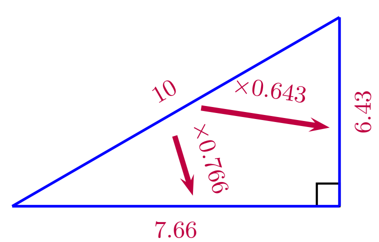
We now consider what if two of the ratios were known on one triangle? Then the solving of such a triangle would be even easier. We just showed and emphasized how to solve the triangle from just one side and one ratio. We now consider the scenario where two of the ratios are known. Suppose we have



This means side b is 64.3% of the hypothenuse, which has size 10, therefore $b = 6.43$ and this means side a is 76.6% of the hypothenuse, which has size 10, therefore $a = 7.66$



Then the to obtain b we need only multiply 10 by the indicated ratio, 64.3% to obtain 6.43 while we obtain a by multiplying 10 by 76.6% to yield 7.66. The triangle with all sides solved would look like this:



Respect the Ratios

The most important idea to understand here is the idea that one ratio and one side is all we need, *and* that *ratios are the key.* The problems posed and solve in this section all were made very doable by knowing one of the ratios between the sides of the triangle, thus a deserved measure of respect and appreciation for rations should be cultivated, as these are a (if not *the*) cornerstone of the study of trigonometry.

All we need is One Side & One Ratio