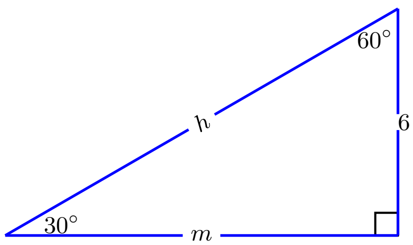
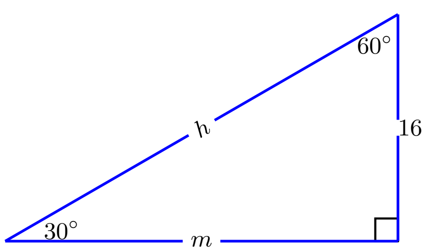
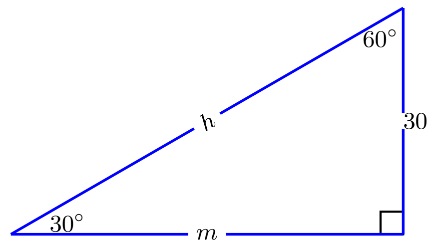
1. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



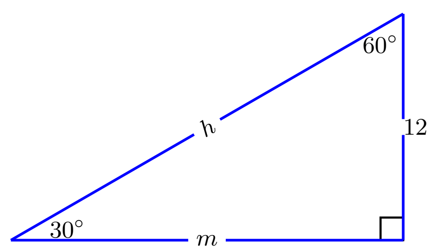

2. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



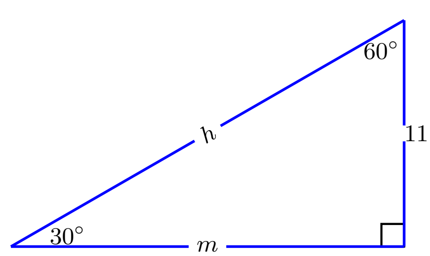

3. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



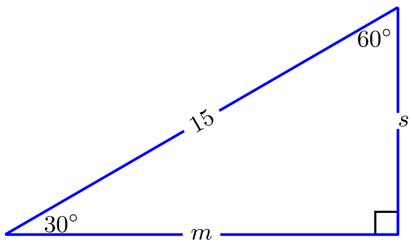

4. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



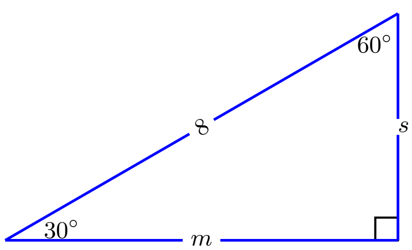

5. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



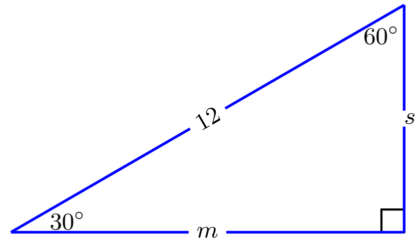
6. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



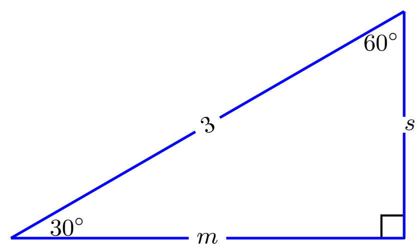

7. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



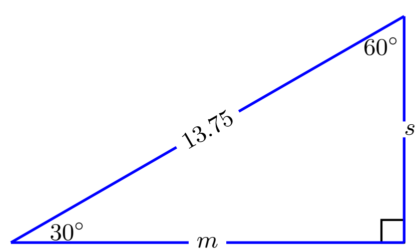

8. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



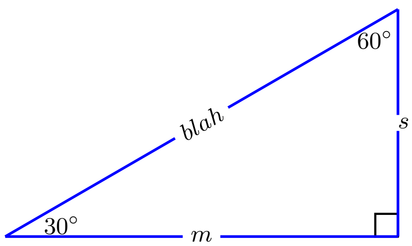

9. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



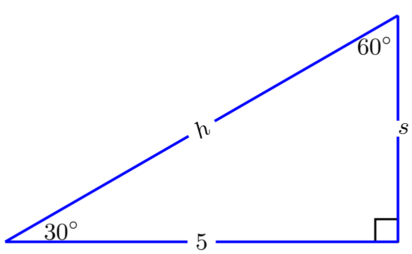

10. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



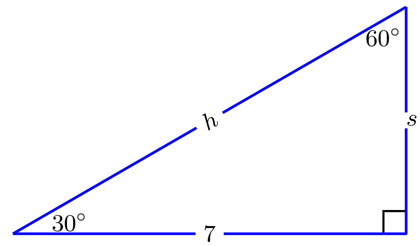
11. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



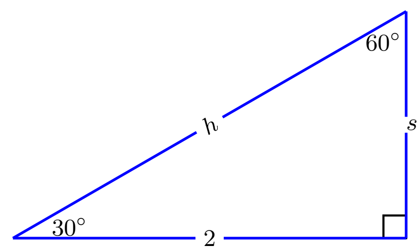

12. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



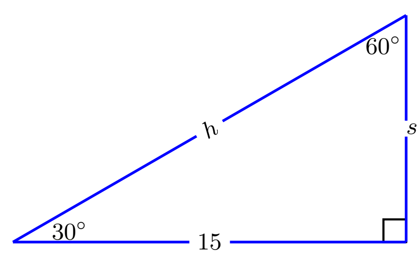

13. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



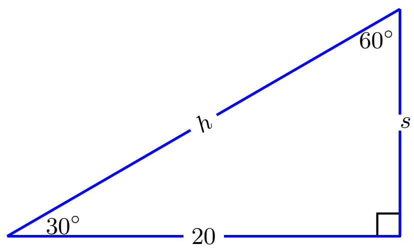

14. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



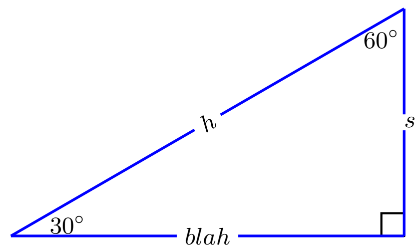

15. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



16. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.




17. Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



## *A SIDE& A RATIO*:

Recall, one of the essential fruits of the Pythagoras Theorem:

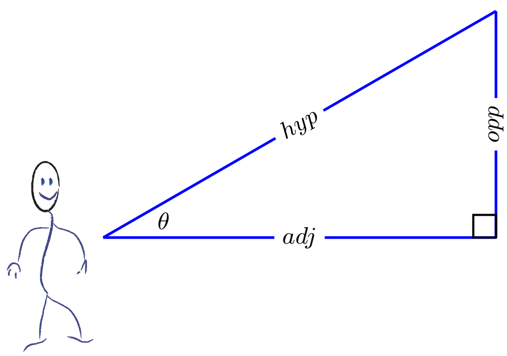
> "*Two sides* of a right triangle are enough to determine *all sides of the triangle*."

The Secret idea from last section makes a similar but different statement:

> "*A side and a ratio* of a right triangle are enough to determine *all sides of the triangle*."

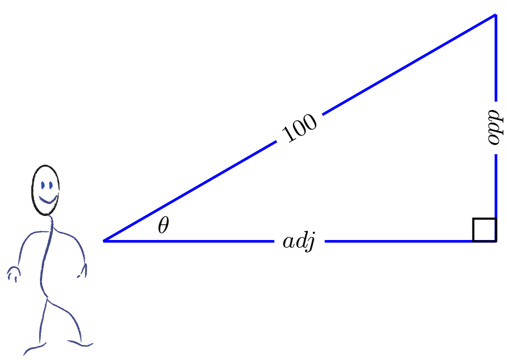
This idea is so very important we take time here to review it, and add some of the common language we will be using throughout this course:

Suppose we pick a corner on a generic right triangle [other than the 90 deg corner]. Suppose, from the selected corner, we labels the sides, adjacent, opposite, and hypothenuse.




Now, suppose we know *one side*, the hypothenuse, is 100 ft. and suppose we know the ratio

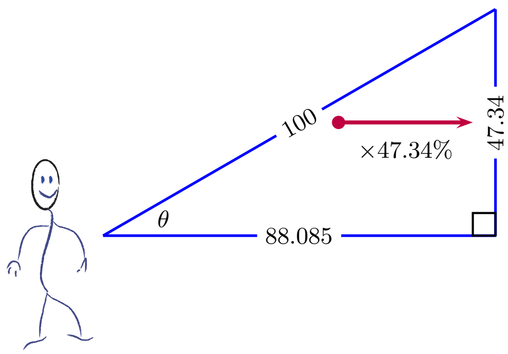
$$\frac{opp}{hyp} = .4734$$




Since the ratio of opposite to hypothenuse is [given] 47.34%, then the opposite side is 47.34% of 100 ft, which is approximately 47.34 ft. Now the hypothenuse side and the opposite side are known, thus we can now use the Pythagoras Theorem to resolve the third side.

$$100^2 = (47.34)^2 + (adj)^2 \quad \text{(PT)}$$
$$10,000 \approx 2241.076 + (adj)^2 \quad \text{(BI)}$$
$$7758.924 \approx (adj)^2 \quad \text{(BI)}$$
$$88.085 \approx adj \quad \text{(BI)}$$

Now, as promised, all sides have been determined from a side & a ratio.

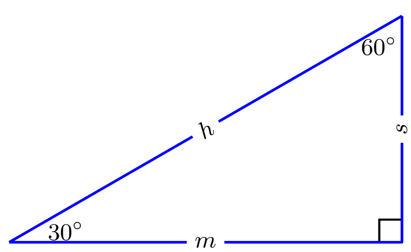



Of course, the elephant in the room is *how did we know the ratio*? or how would one go about finding the ratio? In fact, it's worth imagining, what if we could figure out the ratios for triangles. By now, the ratios should be established and respected and one of the key pieces of the puzzle that aims to solve all sides triangles.

This section and the next one are very special because it is here that we actually learn how to determine, at least for some special triangles, the ratios between the sides. Without further ado.. here it is....

**_THE_ 30-60 Theorem [3060T]:**

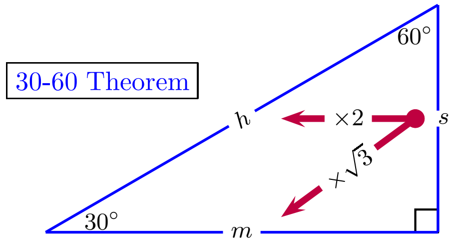
The 30-60 Theorem describes exactly what the ratios are for these very special triangles, namely triangles where the angles are 30-60 and 90 degrees. Note that across from small angles we find small sides, and across from larger angles we find larger sides. Thus, on a 30-60 triangles, the smallest angle is the 30° angle, thus across from it we find the smallest of the sides. Furthermore, 90° is the largest angle in such a triangles, thus, across from it we find the largest side, the hypothenuse. The side across from the 60° angle is neither the largest nor the smallest of the three sides, thus we call it the medium side, and we use $s, m$, and $h$ to label the corresponding sides.




The 30-60 Theorem says that the on such triangles, the small side is *always* half of the the size of the hypothenuse. Or said differently, the hypothenuse is always twice as large as the small. Moreover, the medium side is always $\sqrt{3}$ as large as the small one. In other words, the ratios are as follows:

$$\frac{s}{h} = \frac{1}{2} \qquad \text{and} \qquad \frac{s}{m} = \frac{1}{\sqrt{3}}$$

Yet anther way to put it, to go from the small to the medium we multiply by $\sqrt{3}$, and to go from the small to the hypothenuse we multiply by 2.

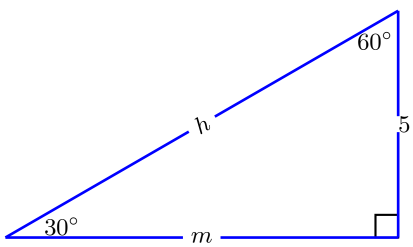


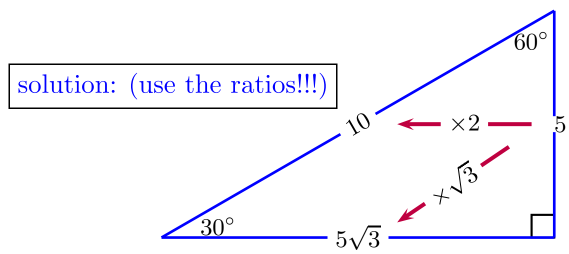

**Example:**

## What is useful for:

**Example:**

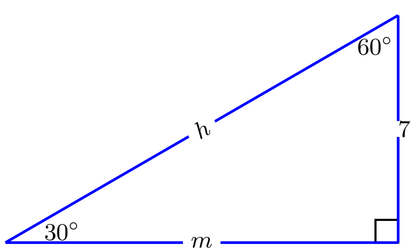
Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.

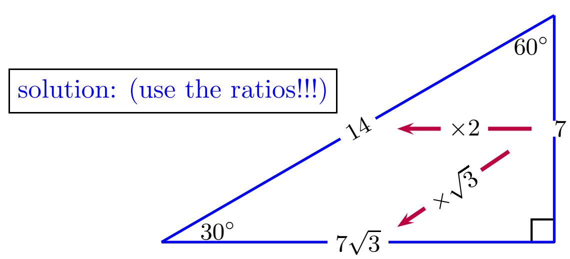



**Example:**

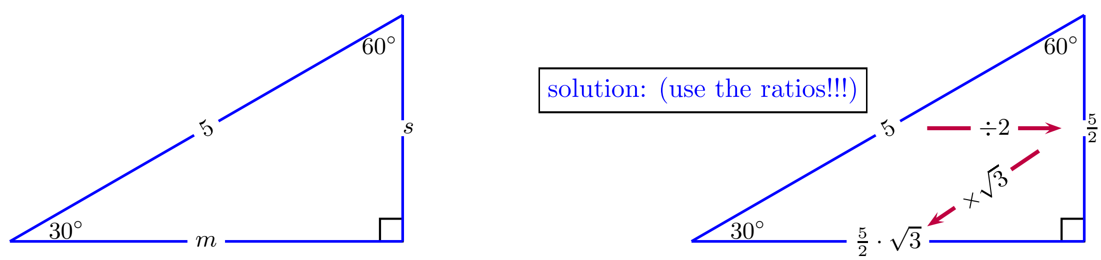
Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.



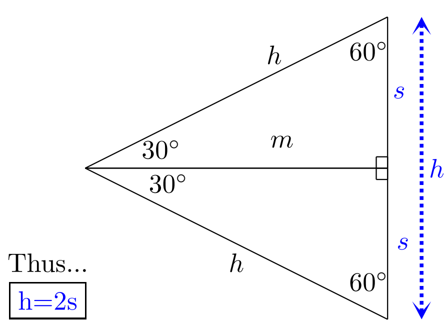
**Example:** In this case the medium[b] side is given.

Observer the following 30-60 Triangle, with only one side given, then determine the other two sides.

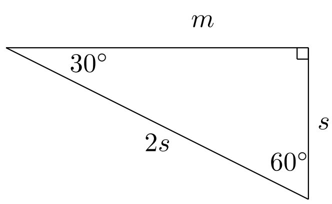



## Make it Yours!

At last, that leaves one immensely important question, *why?* We would like to see a convincing explanation as to why $h = 2s$ and $m = s\sqrt{3}$ on such triangles. The **Key, Essential, & Brilliant Idea** Is to draw a mirror image of this triangle along the medium side.




The result is a large triangle, with all angles the same, 60°, thus all sides the same, $h$. Note one of the sides is also $s + s$, thus $s + s = h$. This is the key idea to understand why $h = 2s$ The medium side is then obtained by using Pythagoras.




| | |
|---|---|
| $(s)^2 + m^2 = (2s)^2$ | Pyth. |
| $s^2 + m^2 = 4s^2$ | alg. |
| $m^2 = 3s^2$ | alg |
| $m = \pm\sqrt{3s^2}$ | SRP |
| $m = \pm s\sqrt{3}$ | alg. |
| $\boxed{m = s\sqrt{3}}$ | in size |