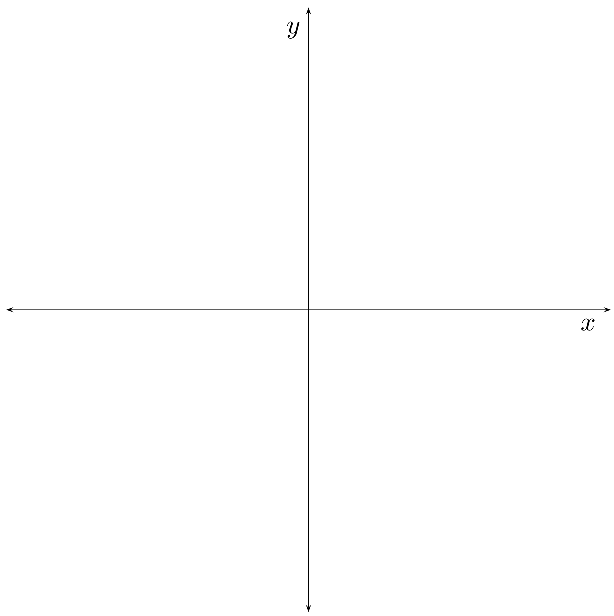
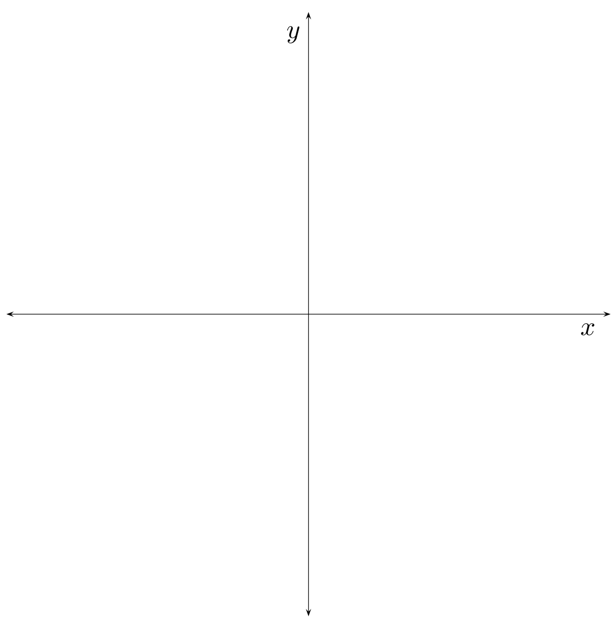
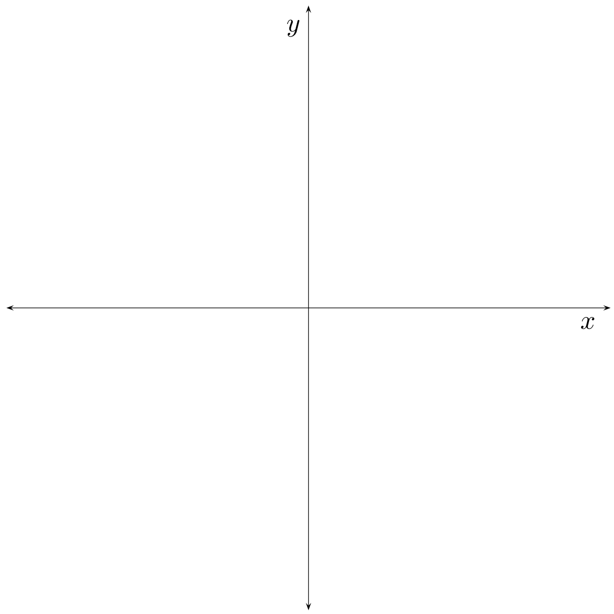
1. *Draw and label* the reference triangle for 210°, using a segment of length 5 , and no calculators.



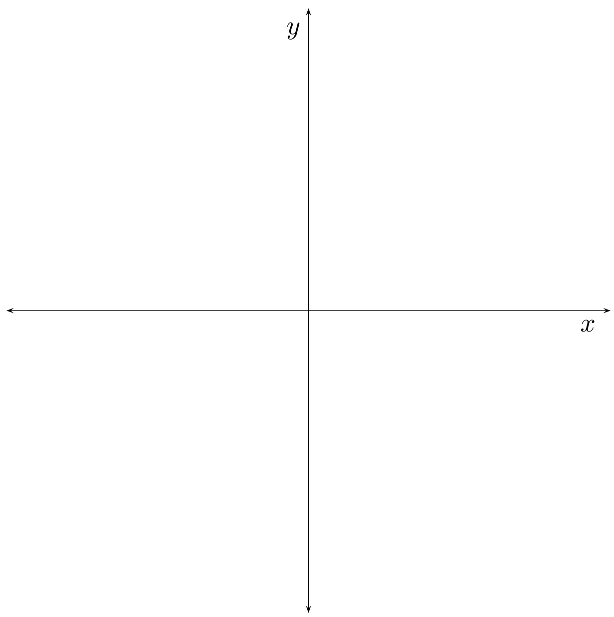
2. *Draw and label* the reference triangle for 330°, using a segment of length 10 , and no calculators.



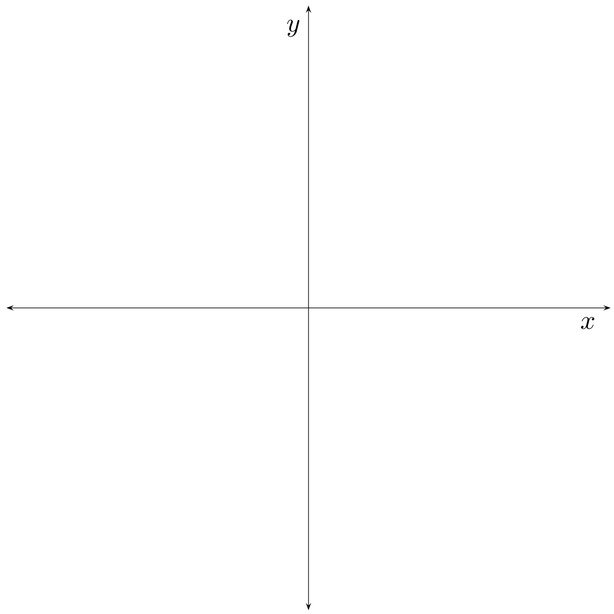
3. *Draw and label* the reference triangle for 30°, using a segment of length 15 , and no calculators.



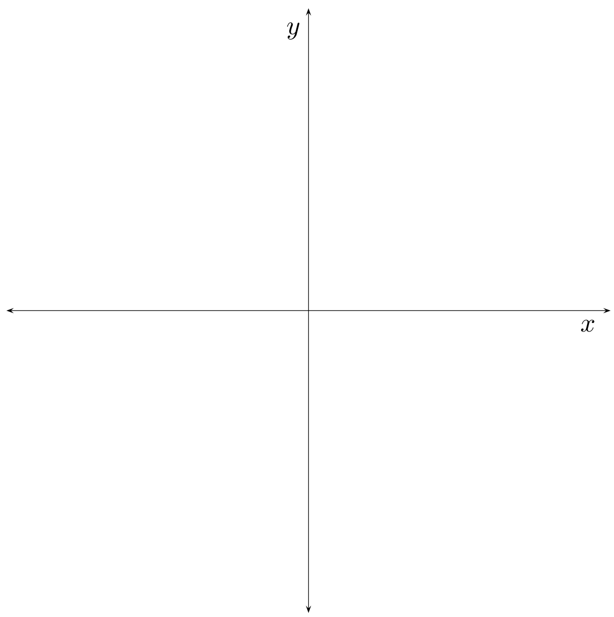
4. *Draw and label* the reference triangle for 150°, using a segment of length K , and no calculators.



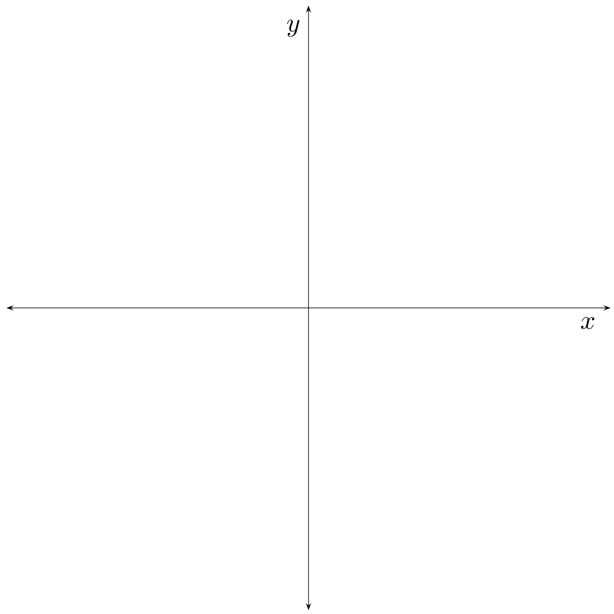
5. *Draw and label* the reference triangle for −150°, using a segment of length R , and no calculators.



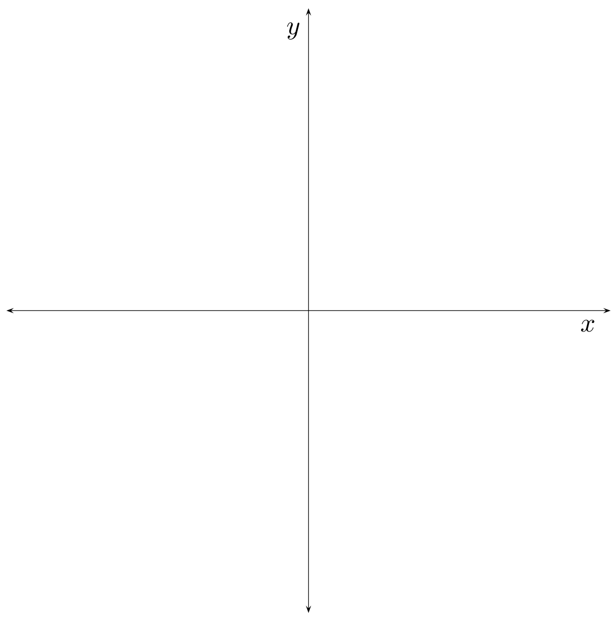
6. *Draw and label* the reference triangle for 120°, using a segment of length 8 , and no calculators.



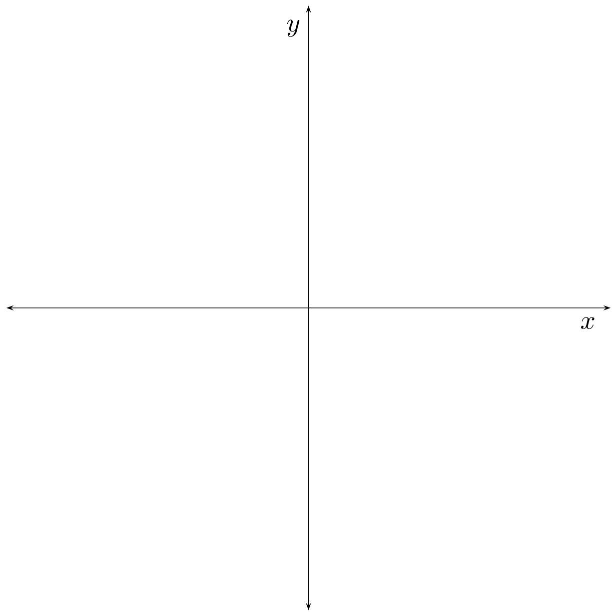
7. *Draw and label* the reference triangle for −120°, using a segment of length 12 , and no calculators.



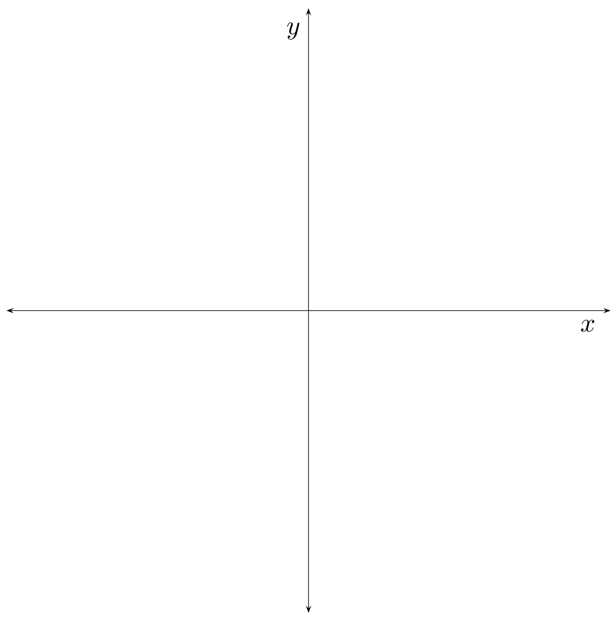
8. *Draw and label* the reference triangle for 390°, using a segment of length 9 , and no calculators.



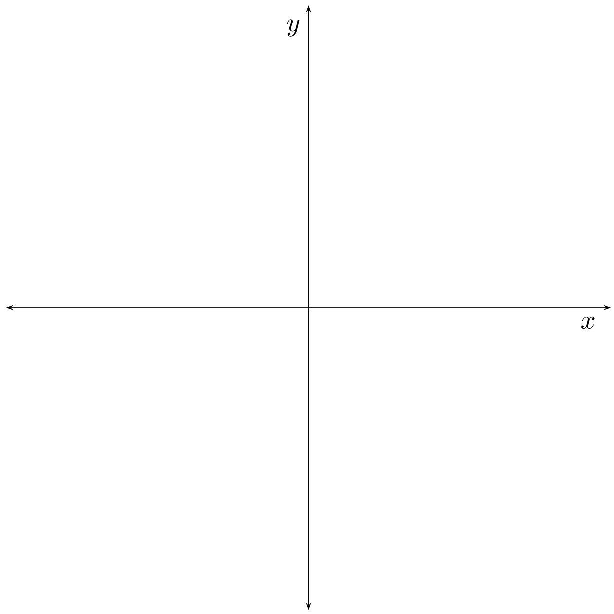
9. *Draw and label* the reference triangle for −210°, using a segment of length 14 , and no calculators.



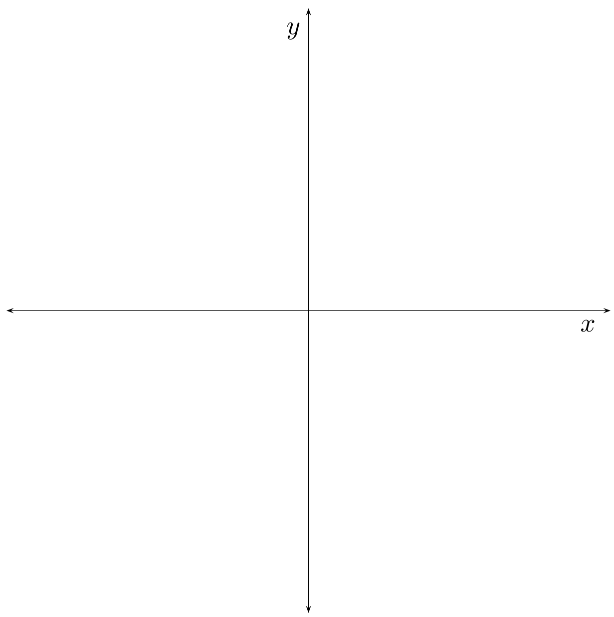
10. *Draw and label* the reference triangle for $-60°$, using a segment of length 6 , and no calculators.



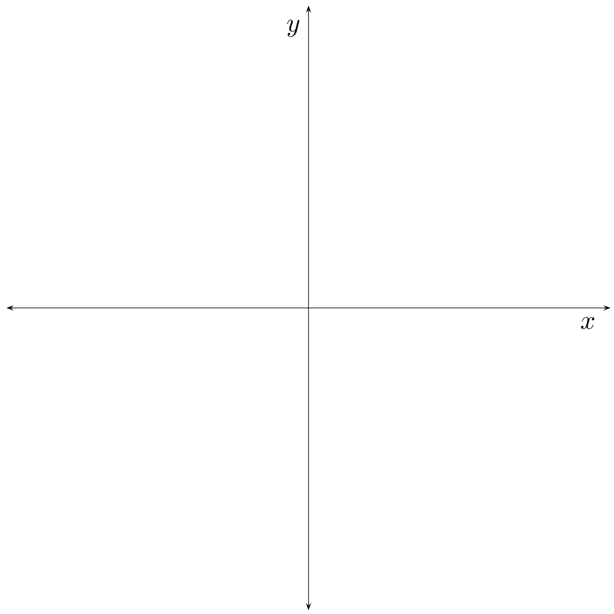
11. *Draw and label* the reference triangle for $750°$, using a segment of length 2 , and no calculators.



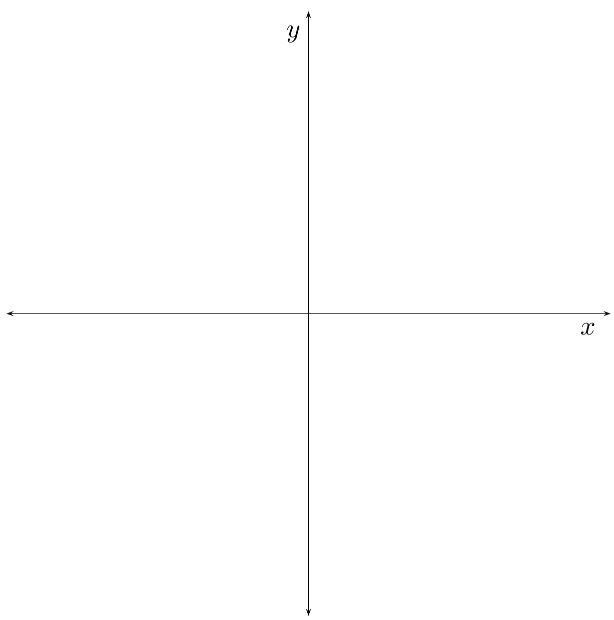
12. *Draw and label* the reference triangle for $-120°$, using a segment of length 1 , and no calculators.



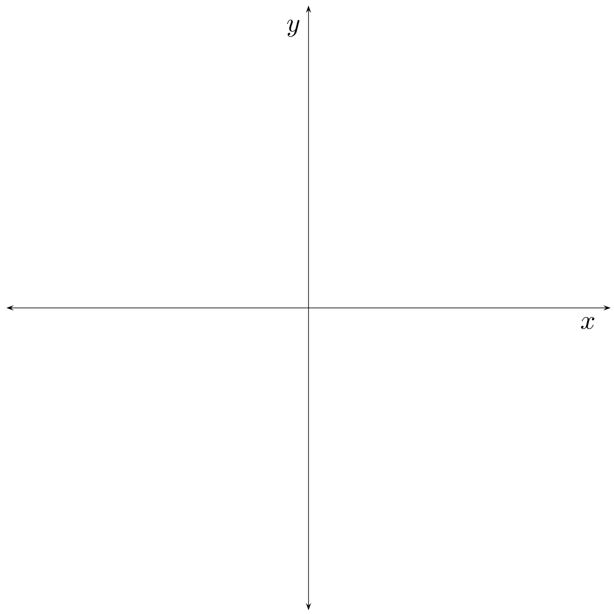
13. *Draw and label* the reference triangle for $-30°$, using a segment of length 7 , and no calculators.



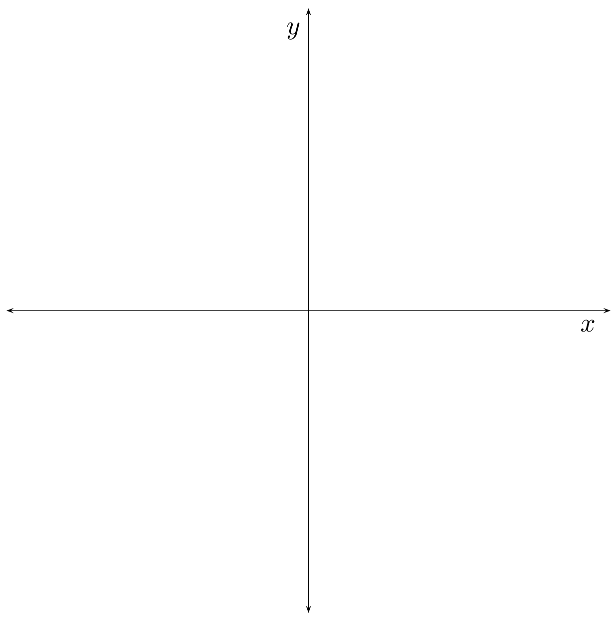
14. *Draw and label* the reference triangle for 45°, using a segment of length 7 , and no calculators.



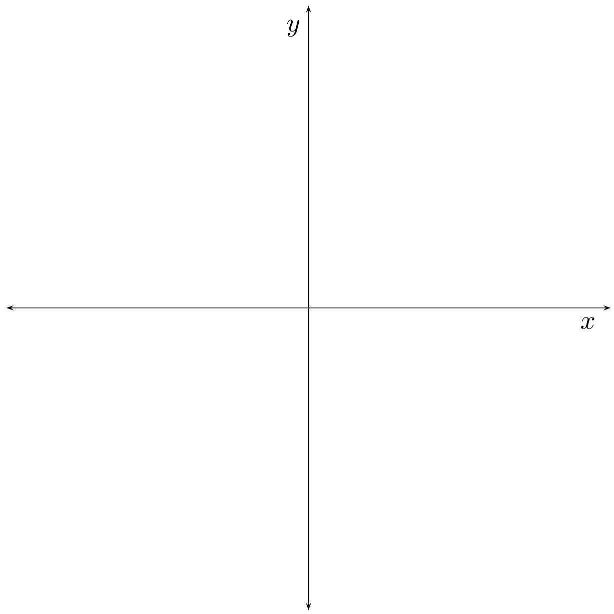
15. *Draw and label* the reference triangle for −135°, using a segment of length 10 , and no calculators.



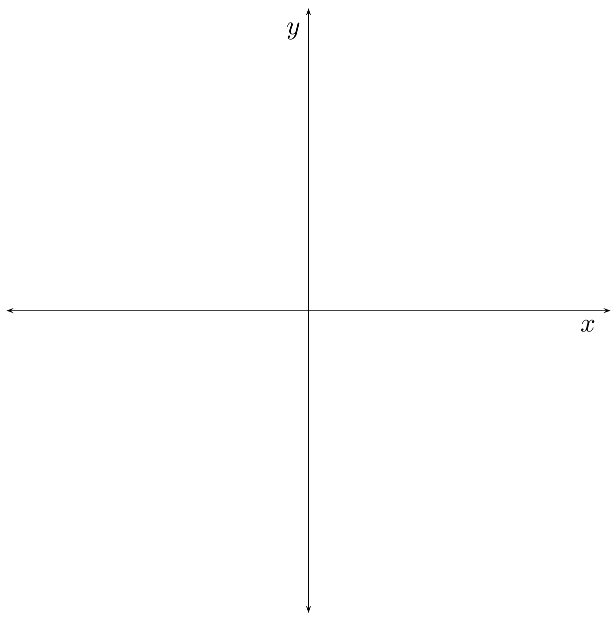
16. *Draw and label* the reference triangle for 135°, using a segment of length 10 , and no calculators.



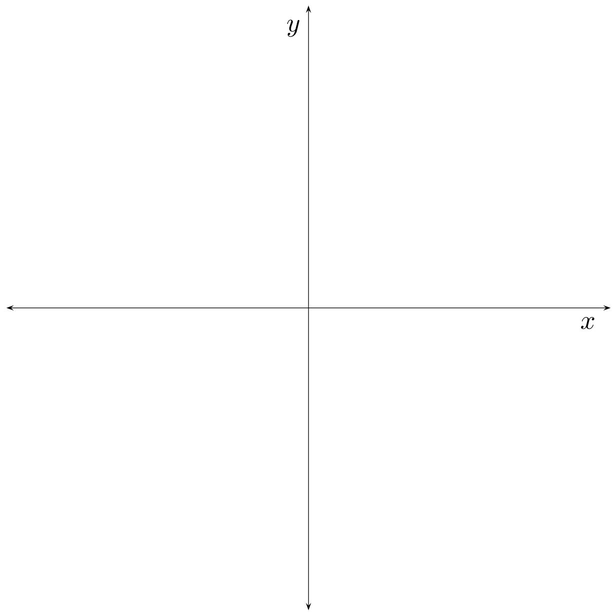
17. *Draw and label* the reference triangle for $-45°$, using a segment of length 7 , and no calculators.



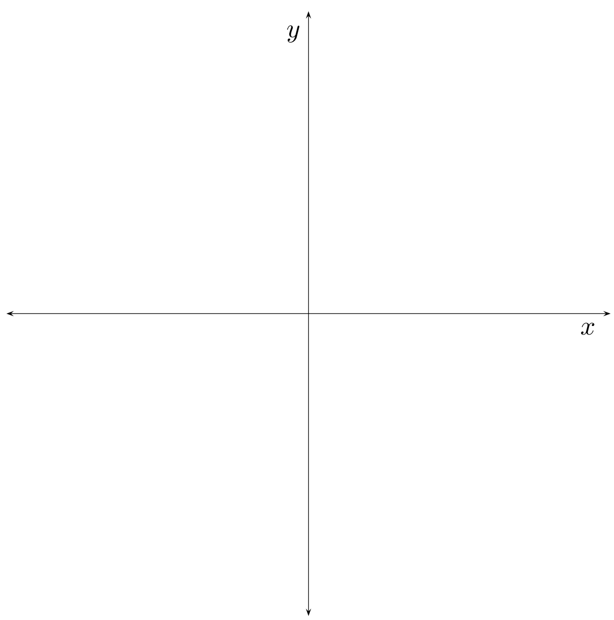
18. *Draw and label* the reference triangle for 225°, using a segment of length 10 , and no calculators.



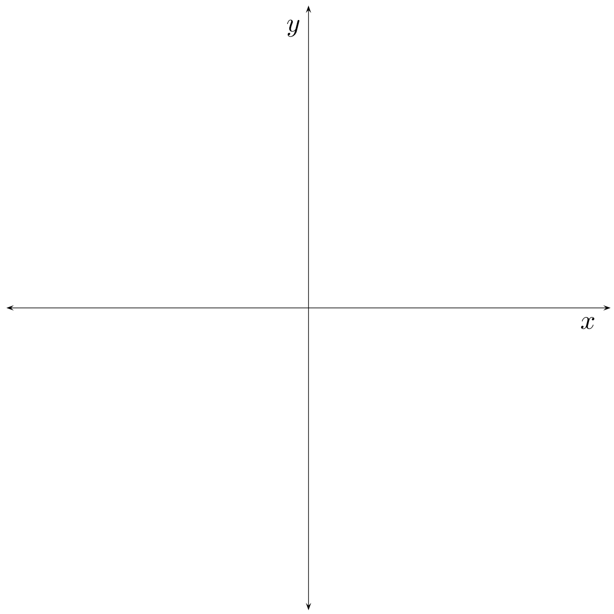
19. *Draw and label* the reference triangle for −225°, using a segment of length 10 , and no calculators.



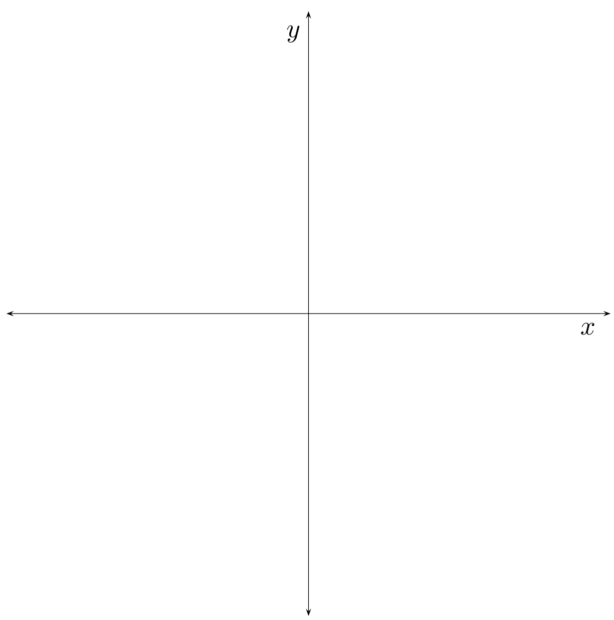
20. *Draw and label* the reference triangle for 495°, using a segment of length 7 , and no calculators.



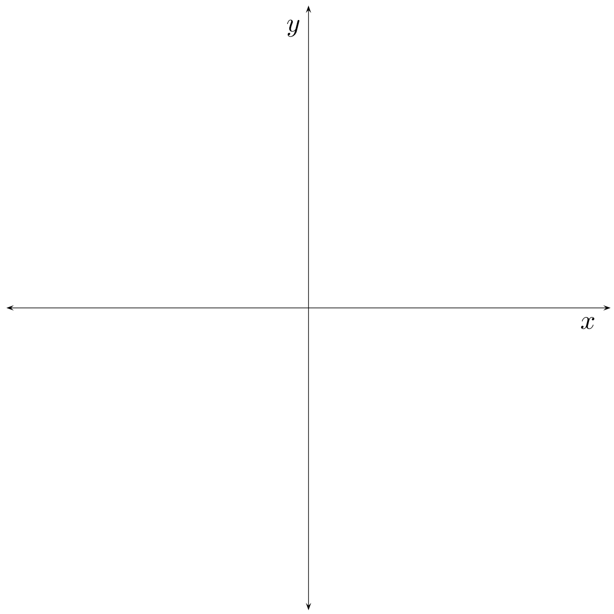
21. *Draw and label* the reference triangle for 315°, using a segment of length 10 , and no calculators.



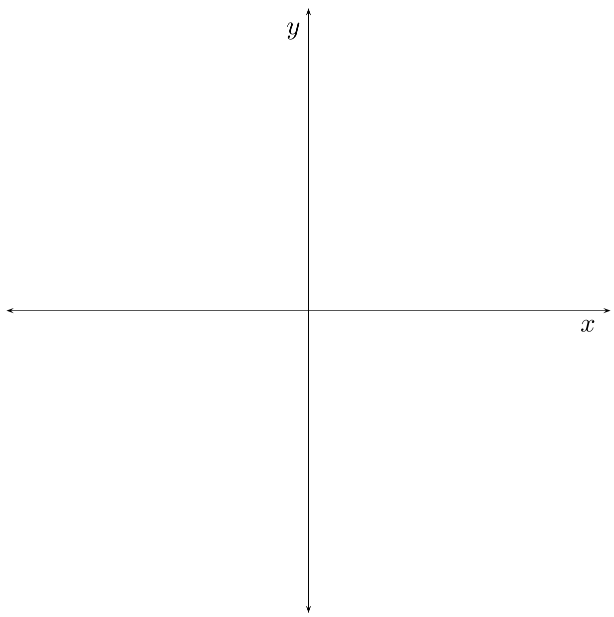
22. *Draw and label* the reference triangle for $-315°$, using a segment of length 10 , and no calculators.



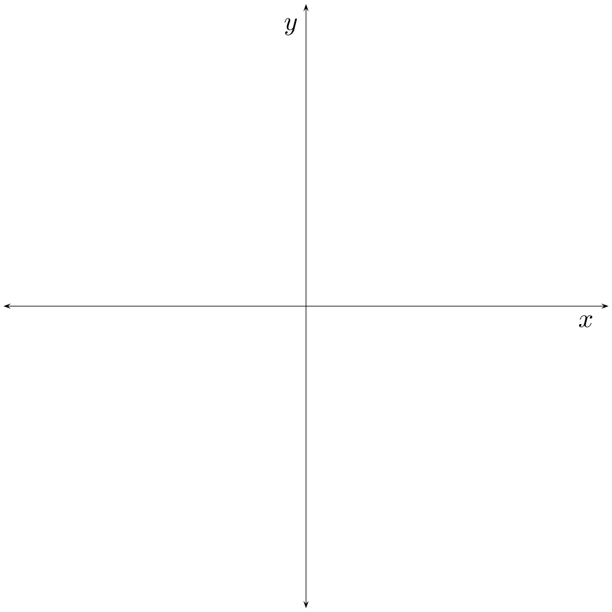
23. *Draw and label* the reference triangle for $405°$, using a segment of length 7 , and no calculators.



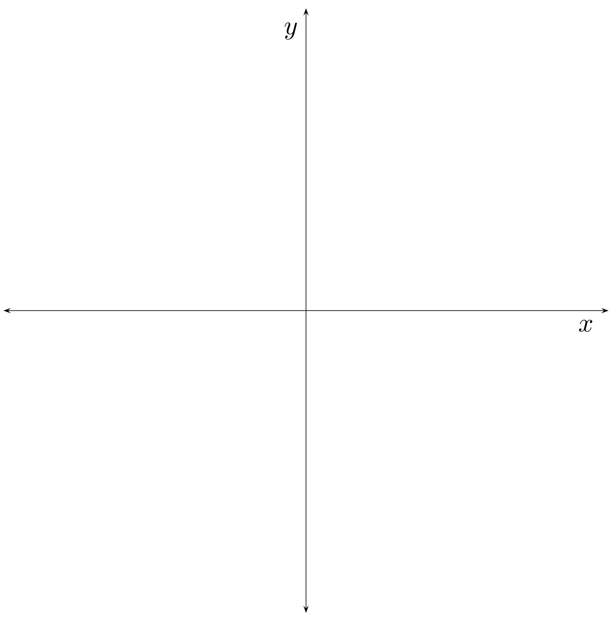
24. *Draw and label* the reference triangle for $-405°$, using a segment of length 10 , and no calculators.



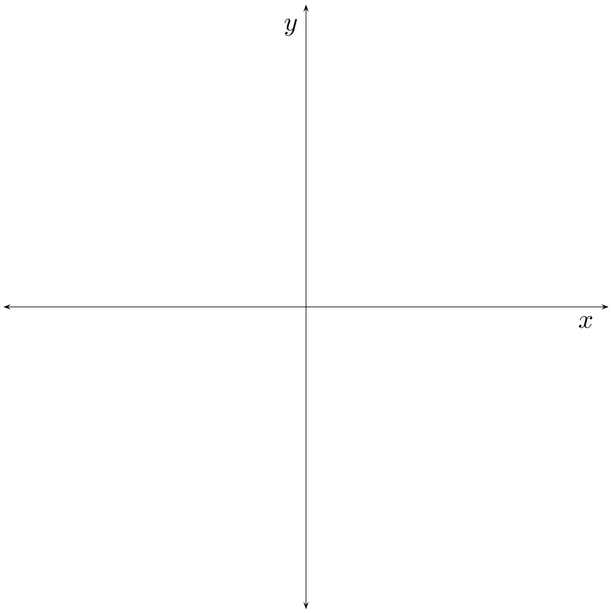
25. *Draw and label* the reference triangle for $675°$, using a segment of length 10 , and no calculators.



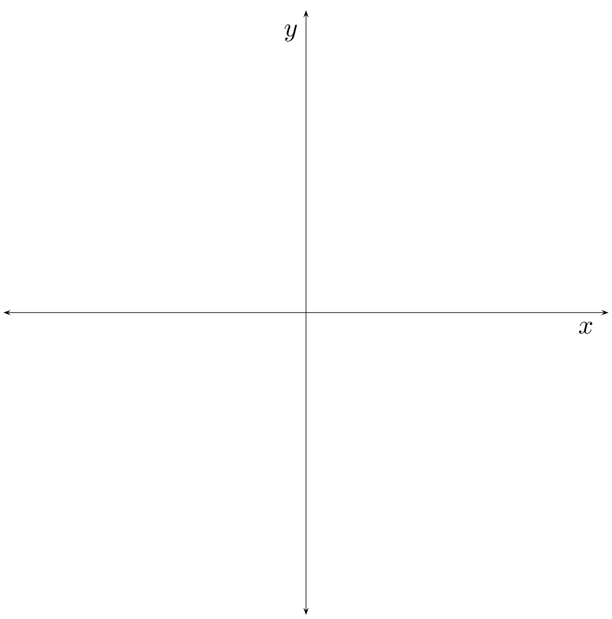
26. *Draw and label* the reference triangle for 270°, using a segment of length 5 , and no calculators.



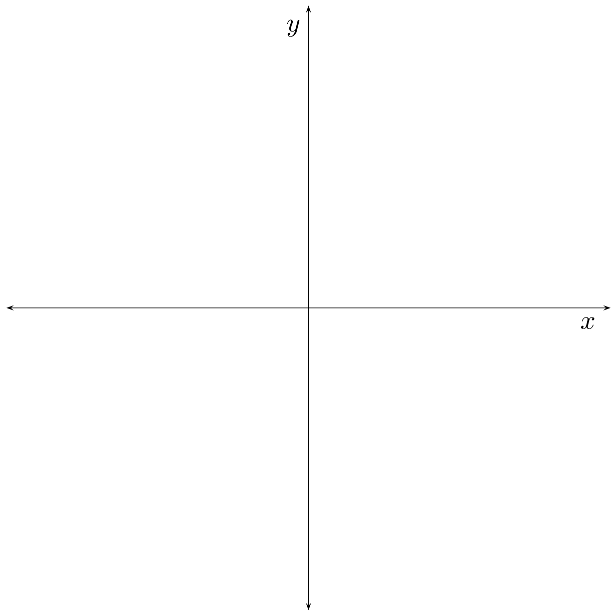
27. *Draw and label* the reference triangle for 180°, using a segment of length 5 , and no calculators.



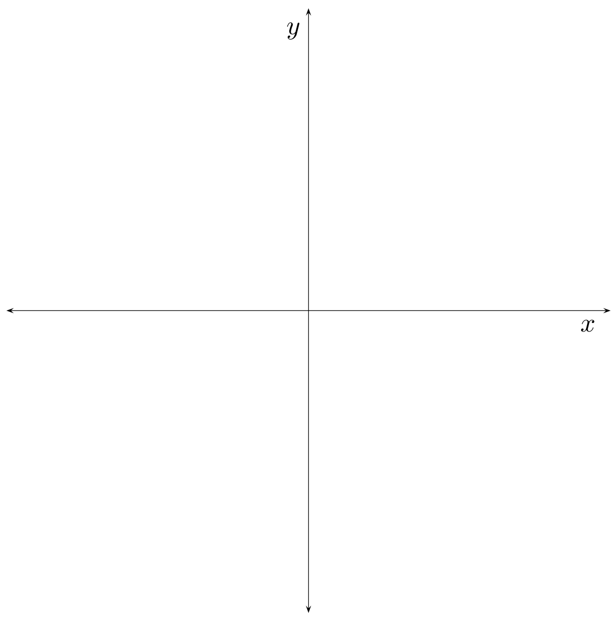
28. *Draw and label* the reference triangle for 450°, using a segment of length 7 , and no calculators.



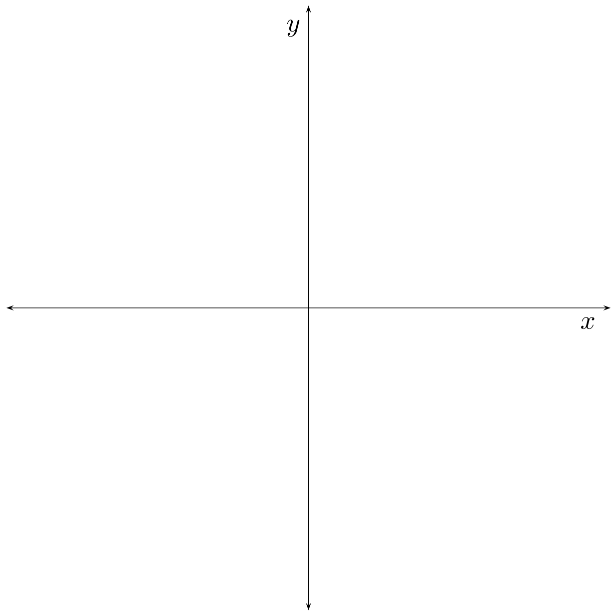
29. *Draw and label* the reference triangle for −270°, using a segment of length 3 , and no calculators.



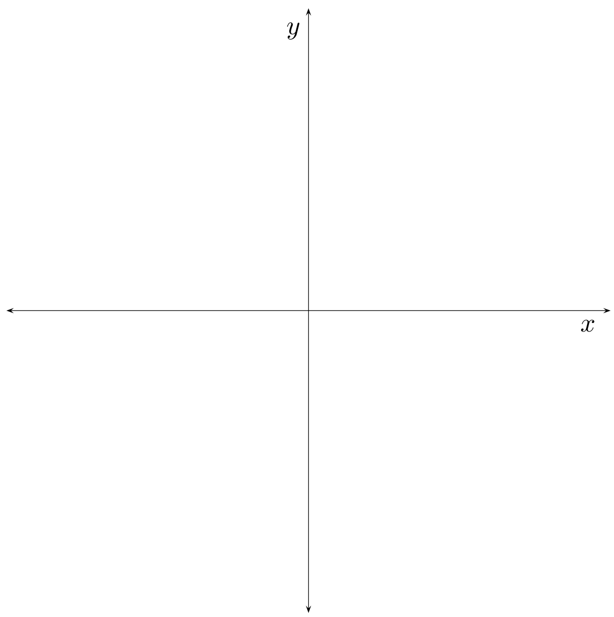
30. *Draw and label* the reference triangle for $-90°$, using a segment of length 8 , and no calculators.



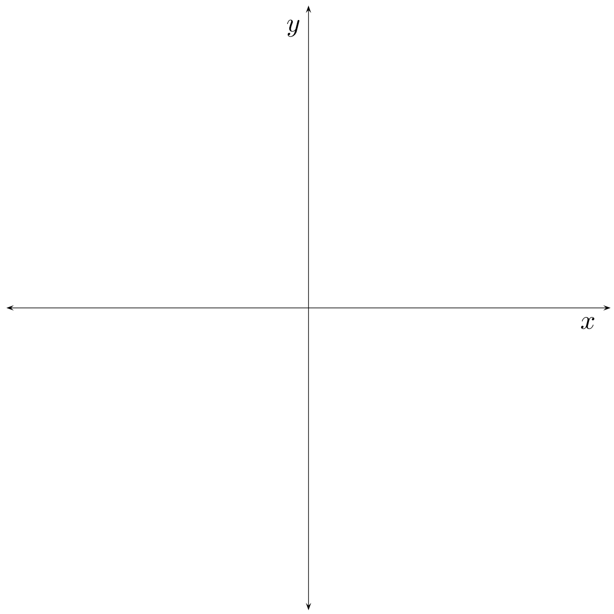
31. *Draw and label* the reference triangle for $0°$, using a segment of length 8 , and no calculators.



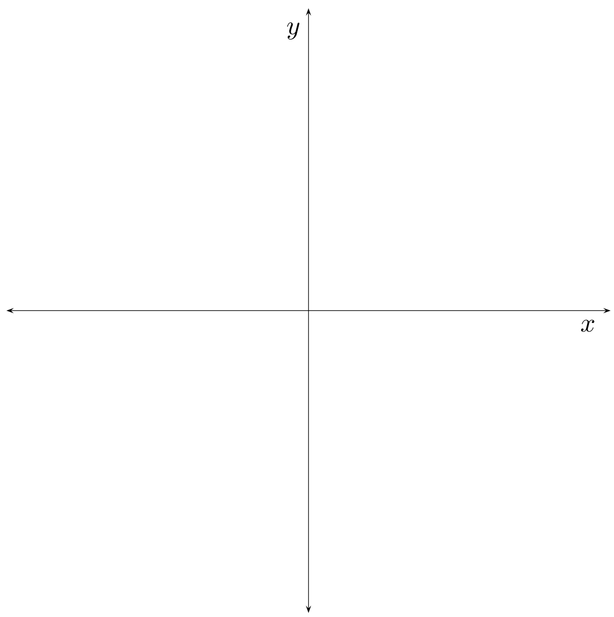
32. *Draw and label* the reference triangle for 270°, using a segment of length 5 , and no calculators.



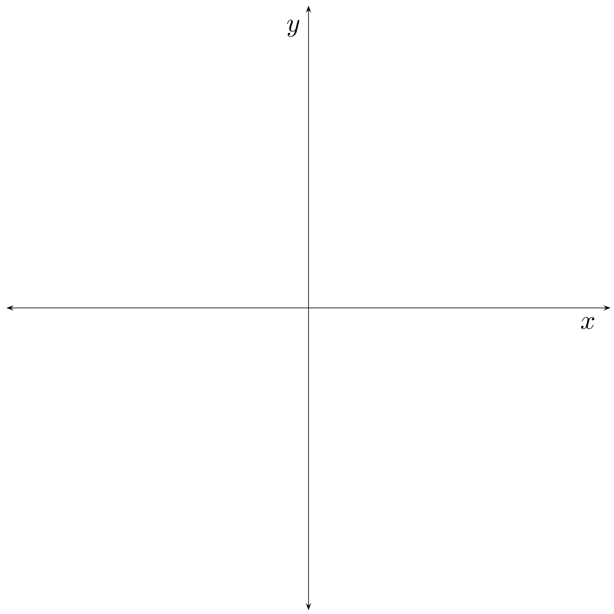
33. *Draw and label* the reference triangle for −450°, using a segment of length 5 , and no calculators.



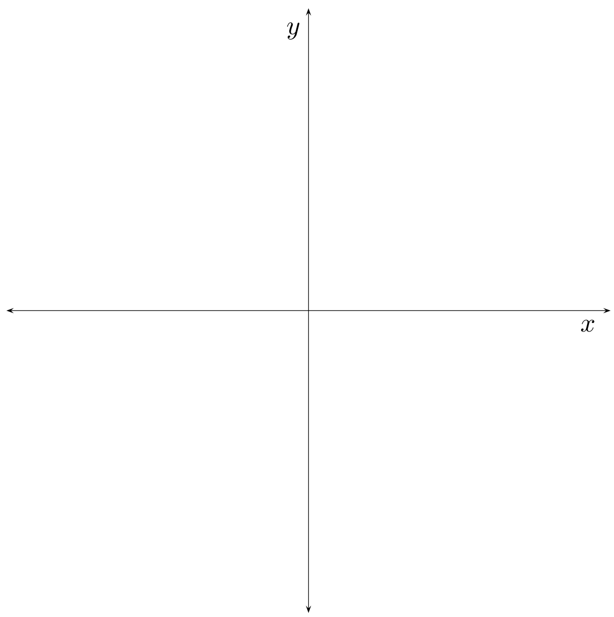
34. *Draw and label* the reference triangle for $-540°$, using a segment of length 5 , and no calculators.



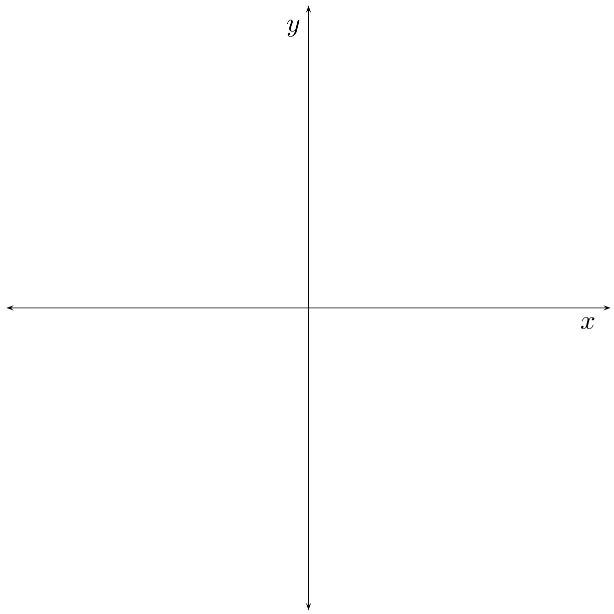
35. *Draw and label* the reference triangle for 720°, using a segment of length 5 , and no calculators.



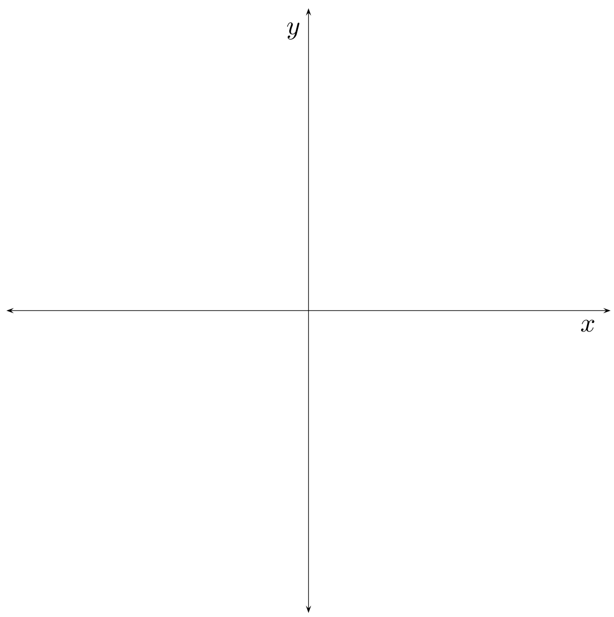
36. *Draw and label* the reference triangle for $-540°$, using a segment of length 5 , and no calculators.



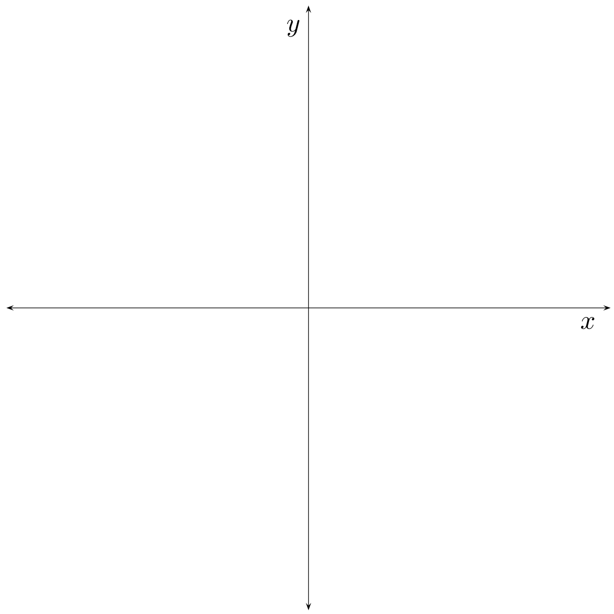
37. *Draw and label* the reference triangle for $-630°$, using a segment of length 5 , and no calculators.



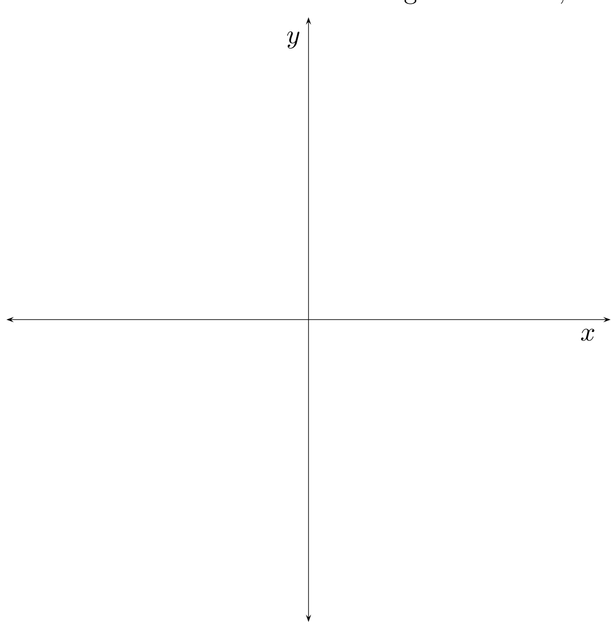
38. *Draw and label* the reference triangle for 360°, using a segment of length 5 , and no calculators.



39. *Draw and label* the reference triangle for −360°, using a segment of length 5 , and no calculators.

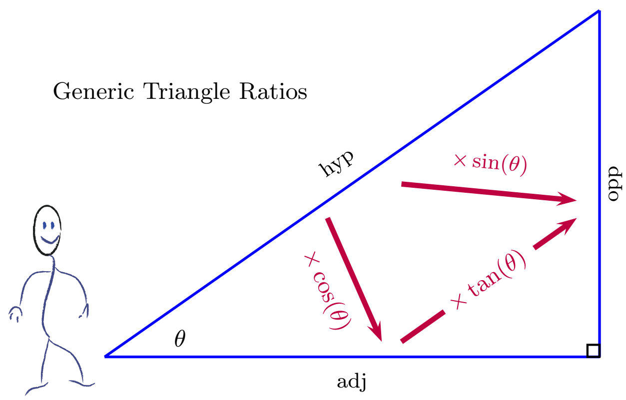


40. *Draw and label* the reference triangle for $-810°$, using a segment of length 5 , and no calculators.



The MAIN IDEA:

The last couple sections are the cornerstone of trigonometry. These established the foundation for this chapter and for essential trigonometric ideas. A deep and meaningful understanding the trigonometry will depend on a deep and profound understanding of the definitions of the trigonometric functions. In the last section, we introduced these functions. We described them as the ratios of sides of a right triangle. We summarize these ideas with this:



SOH-CAH-TOA

$$Sin(\theta) = \frac{Opp}{Hyp}$$

$$Cos(\theta) = \frac{Adj}{Hyp}$$

$$Tan(\theta) = \frac{Opp}{Adj}$$

With these functions and our ability to calculate them using some calculator, we could effectively made good on one of the essential promises of trigonometry, to be able solve any right triangle given enough information.

However, once this is accomplished, the trig functions take a life of their own. They become important in their own right, and the become in some sense bigger than the original problem they were made to solve. In this section, we turn such corner, we begin to study the trig function, not so much to solve right triangles, but simply to study their properties and behavior. The usefulness and implications of these functions are much bigger than this course. We will be content, and we will feel fortunate just to get to know these function just a tiny little bit.

For example, at this point we are well versed with the meaning of

$$\sin(23^\circ)$$

as the ratio of opp/hyp on the corresponding right triangle, we never did ponder the meaning of something like

$$\sin(275^\circ)$$

In fact, it may seem a little wild to utter such words since our right triangles usually contained angles measuring less than 90°. Said differently, we defined our functions for little angles, angles between 0 and 90°, but as implied earlier, the function grew to mean much bigger and deeper ideas than that. Specifically, we now begin the study of *reference triangles* which will eventually lead to the extension of the trig functions to angles larger than 90° or even negative angles. Moreover, this will be consistent with the ideas we have thus far.

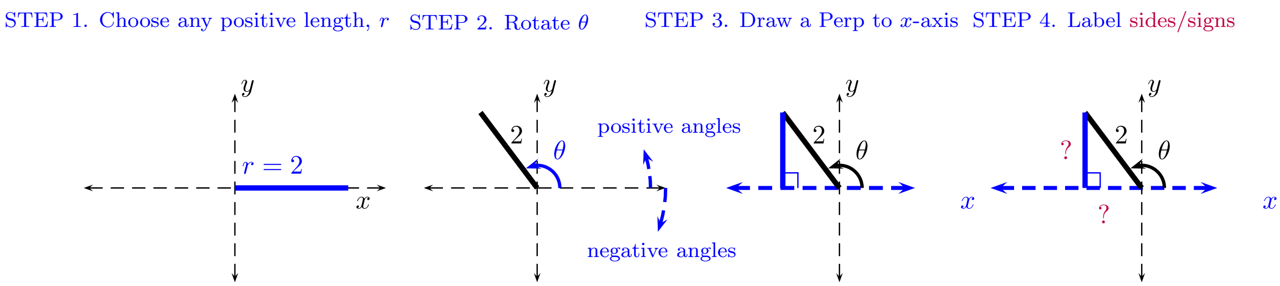
Talk is cheap! Let us begin to define the very important concept of a *reference triangle* and the idea that *every angle has one [at least].*

Without further ado, here is the definition of a *reference triangle.*

EVERY ANGLE has a REFERENCE TRIANGLE:

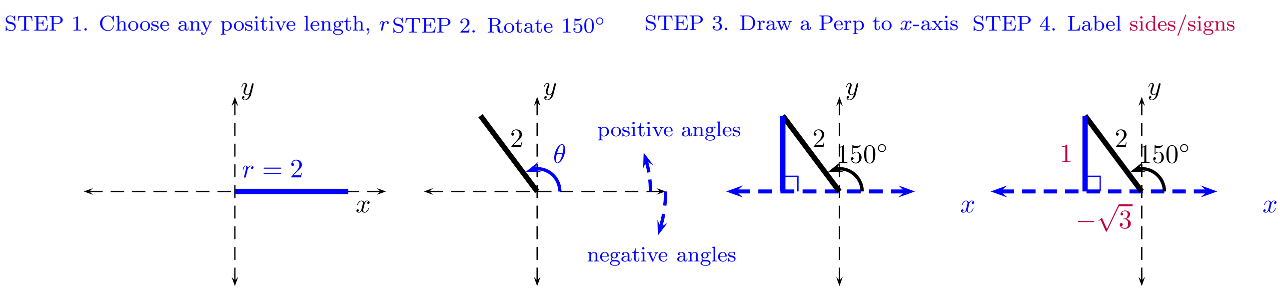
Given any angle, θ:

1. We choose a line segment of any positive length, r, and place the segment one endpoint on origin of the $xy-plane$, the other endpoint on the positive side of the x-axis.
2. Rotate the segment, maintaining the endpoint at the origin fixed, θ units (degrees or radians). Rotate positive angles counterclockwise, rotate negative angles clockwise.
3. From the endpoint of the segment not on the origin, draw a perpendicular *to the x-axis*. This will create a triangle or an almost triangle. This is called and defined to be *the reference triangle for angle θ*
4. The fourth step is to label the sides of the *reference triangle*, including signs where appropriate. This means, up is positive down is negative, right positive, left is negative, as usual, for now we will keep the r positive. In later chapters, we will have a meaningful interpretation for negative r's. One should note this last step will be possible, with our current level of knowledge, only for famous angles, such as $0°, 30°, 45°, 60°$, etc...

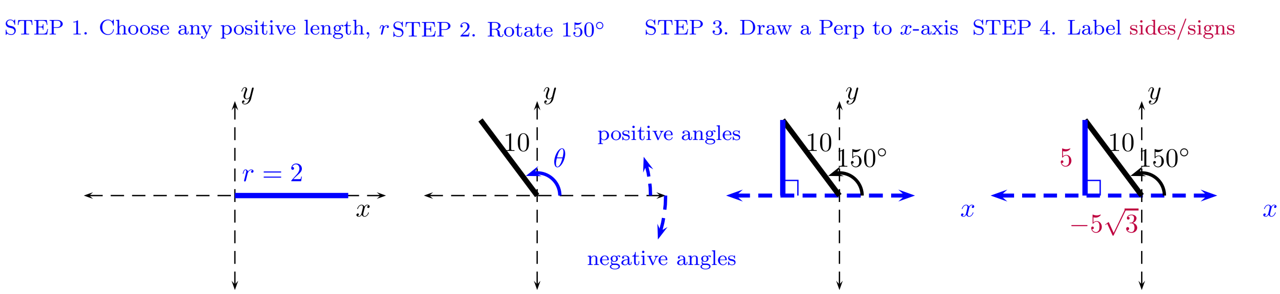


Examples:

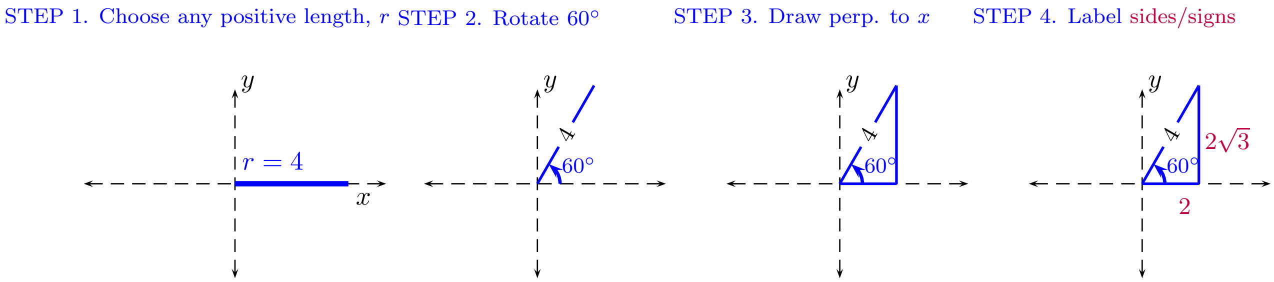
Example: Draw the reference triangle for $150°$ with a line segment of length 2.



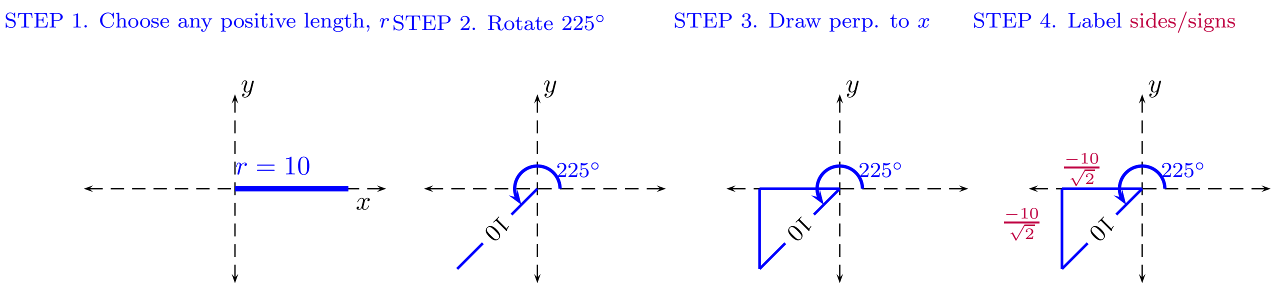
Example: Draw the reference triangle for $150°$ with a line segment of length 10.



Draw the reference triangle for 60°. Use a line segment of length 4 units, and do not use calculators.



Draw the reference triangle for 225°. Use a line segment of length 10 units, and do not use calculators.



Finally, it should be noted that the hypothenuse is always, by definition kept positive, and that it's length can be any positive real number, thus in reality every angle has many reference triangles, depending on how big the initial segment is drawn.