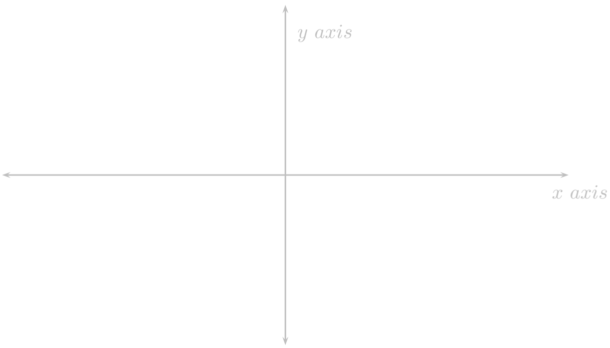
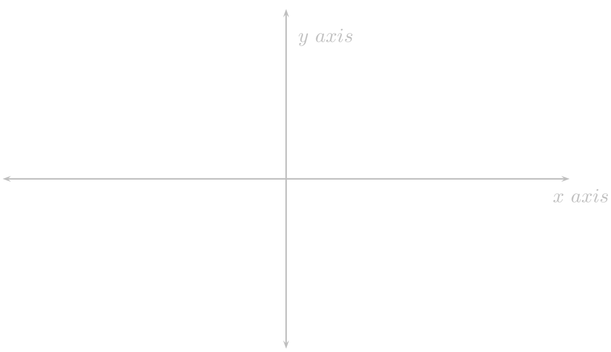
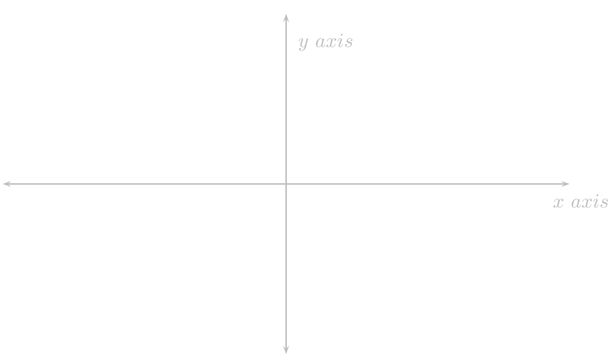
1. 120°



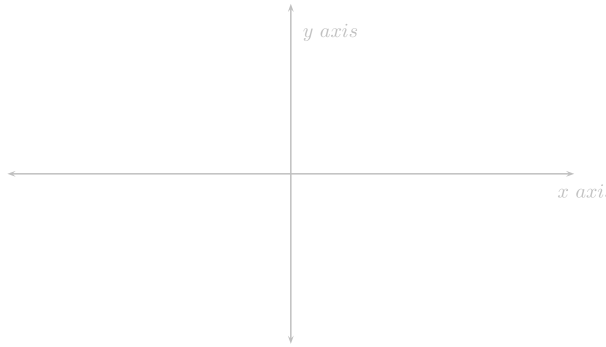
2. 150°



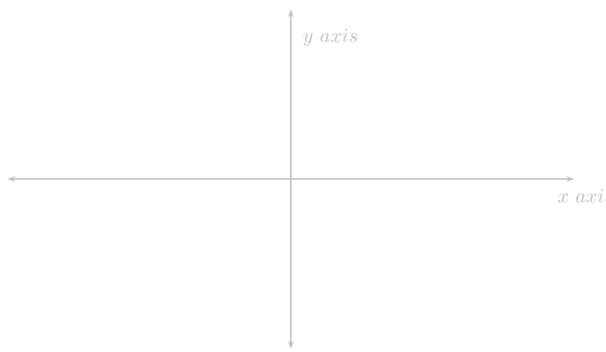
3. −120°



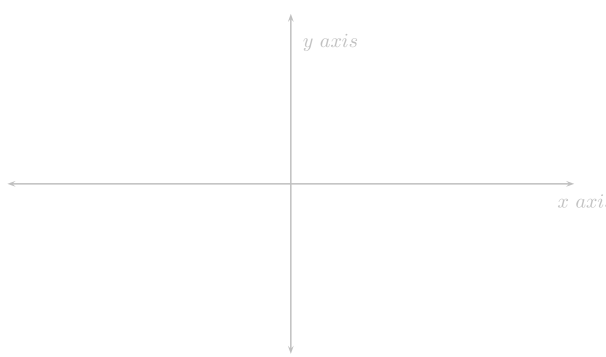
4. 330°



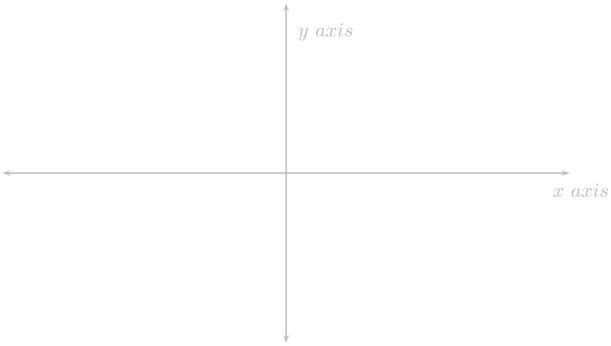
5. 135°



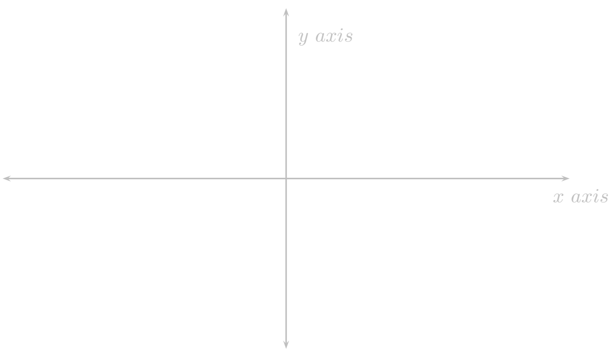
6. 225°



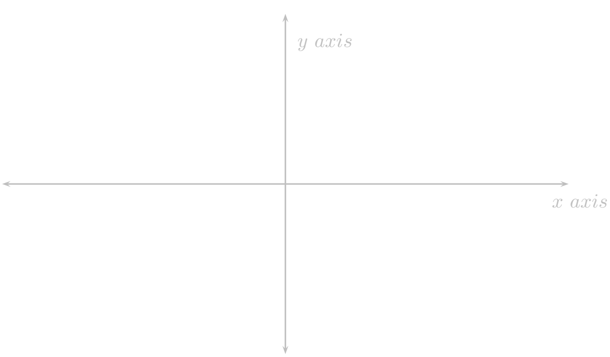
7. 315°



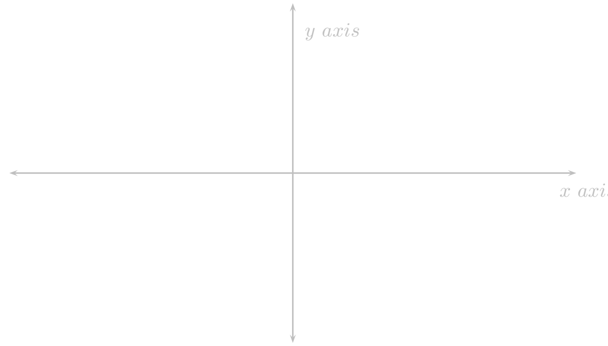
8. 30°



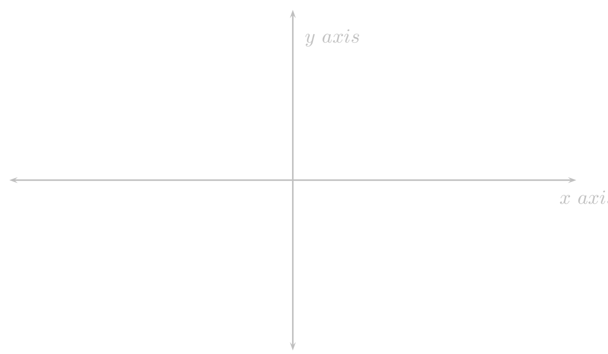
9. $-30°$



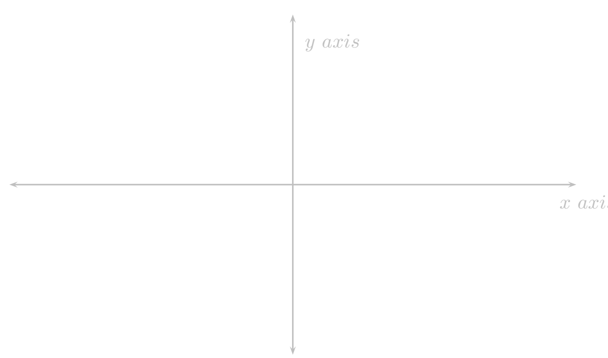
10. 690°



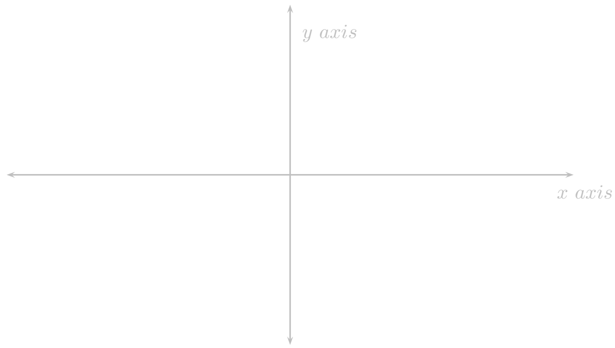
11. $-330°$



12. $-210°$



13. 180°



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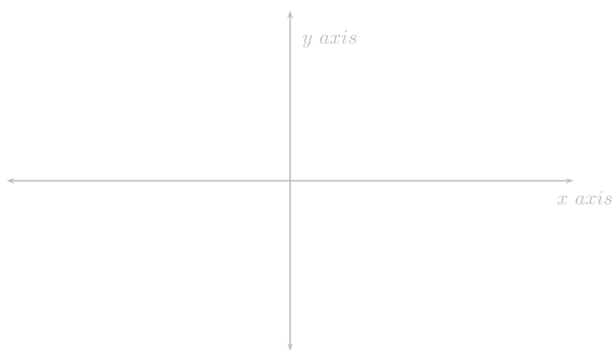
14. $-180°$



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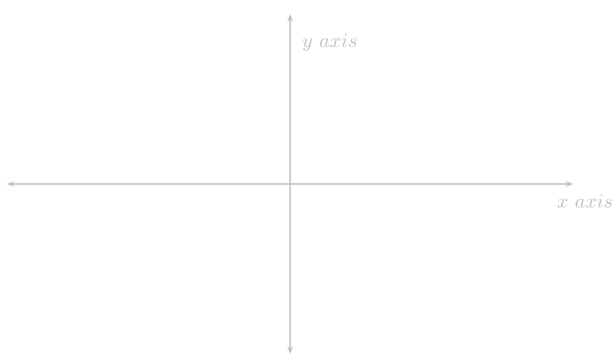
15. $360°$



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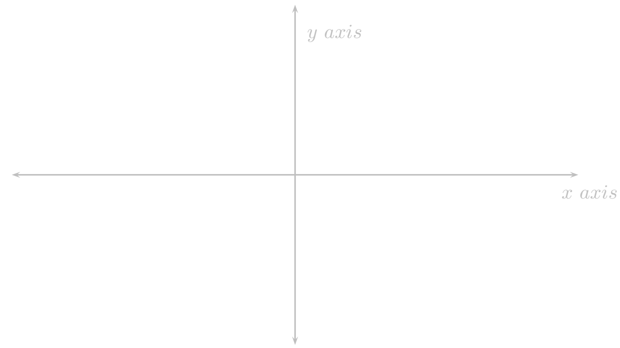
16. $-360°$



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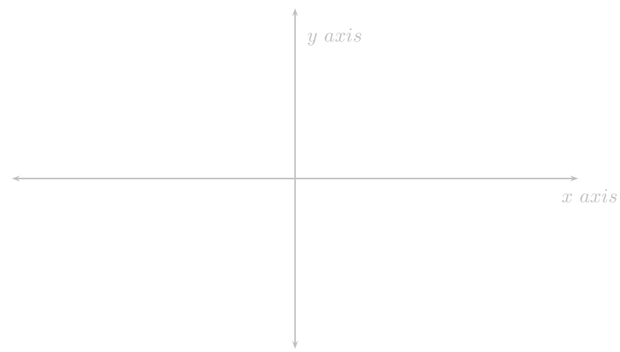
17. $90°$



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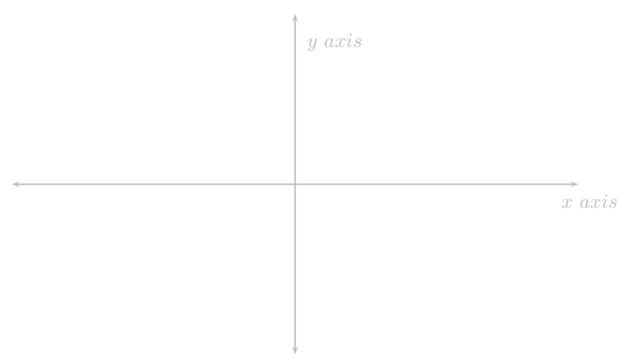
18. $-90°$



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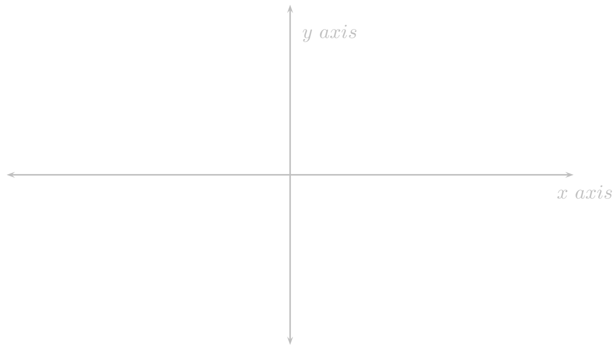
19. $270°$


y axis
x axis

THE REST of the SINE and COSINE Functions:

This section is yet another immensely critical milestone in our journey to understanding all that is trigonometry.

It is here and now that we extend these functions for angles larger than 90° or smaller than 0° (when possible). We will learn how to evaluate them, graph them, use them to solve triangles, solve equations with such functions in them, and we will study some of their most famous and celebrated properties. Thus, once again, the importance of this section can not be overstated.

There are six very famous trig functions, sine, cosine, tangent, cotangent, cosecant, and secant. Sine and Cosine are personal favorites, in some way, all other functions are composed of variations of these. Thus, we will focus most of our attention on these two functions for now. Once comfortable with these, we will move on to the other four functions. Before we dive into the small details, we take a big-picture perspective on what these functions are. *These functions are ratios.* For any given angle, we construct a reference triangle. These functions are, by definition, ratios of certain sides of the corresponding reference triangle. Once angle is given and the reference triangle constructed and labeled, we stand at the origin and observe which of the sides of the triangle is *adjacent*, which side is *opposite*, and which side is the *hypothenuse*. The cosine function is defined to be the ratio of *adjacent/opposite*, denoted $\cos\theta = \frac{adj}{opp}$.It should be clear which side of the reference triangle is the hypothenuse, it's across from the 90° angle. The opposite side and the adjacent sides depend very much on our point of reference. The definitions of all our trig functions all assume the point of reference used is the origin. That is, once we use the origin as a point of reference, there is only one side which is opposite. The other sides are also easily identified, they are the adjacent and the hypothenuse.

We state the definition formally:

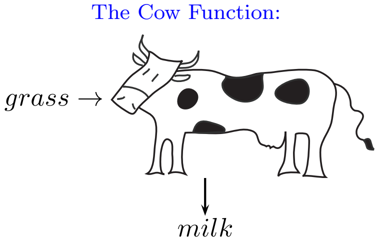
DEF: of $\sin\theta$ and $\cos\theta$

For any angle, θ, we construct the *reference triangle* for θ using any positive length r for the hypothenuse. We label the sides, 'hypothenuse', 'adjacent', and 'opposite', accordingly. $\sin\theta$ is defined as the ratio of the sides of the reference triangle:

$$\sin\theta = \frac{opposite}{hypothenuse} \qquad \cos\theta = \frac{adj}{hypothenuse}$$

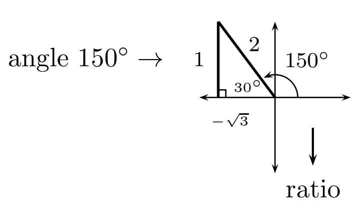
In layman terms, you can think of a function as a *cow*. The cow eats *grass* and produces *milk*. In a similar way, the cosine function 'eats' angles and produces *ratios*.

The Cow Function:



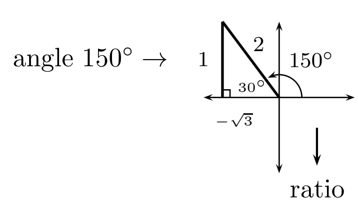
cow(*grass*)=*milk*

The Cosine Function: adj/hyp



$\cos(150°) = \frac{-\sqrt{3}}{2}$

The Sine Function: opp/hyp



$\sin(150°) = \frac{1}{2}$

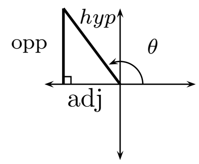
THE REST of the Other Trig FUNCTIONS:

We are now ready to meet the other famous trigonometric functions, tangent, cotangent, secant and cosecant. These are very similar to sine and cosine in the sense that they too describe ratios of sides of a corresponding reference triangle. For any given angle, θ, we construct a reference triangle, then we define the functions as follows:

DEF: of the SIX trig functions

angle θ $\rightarrow$

reference triangle for θ



ratio $\rightarrow$

$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

$\csc\theta = \frac{hyp}{opp}$ $\quad \sec\theta = \frac{hyp}{adj}$ $\quad \cot\theta = \frac{adj}{opp}$

It is important to direct your attention momentarily to the denominators on each of the 6 ratios. Sine and cosine each have 'hyp' as the denominator. Keep in mind the hypothenuse is a positive length, r, of the segment chosen to construct the reference triangle. By definition, such $r = hyp$ will never be zero. Thus, such ratios will be well defined for all possible reference triangles and in turn for all possible angles, θ. Said differently, the sine and cosine functions are defined for all possible real angles. The same can not be said of any of the other functions. Consider, for example, the case when $\theta = 90°$. In such case, if $hyp = 2$ then opposite side is 2 and the adjacent is 0. Since the ratio $\frac{2}{0}$ is not a real number, thus the tangent of 90° does not exist as a real number. There are many angles (all which end up in an 'almost triangle' reference triangle) for which the other four functions are not defined.