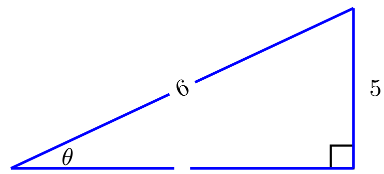
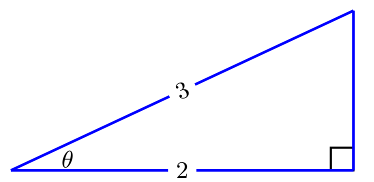
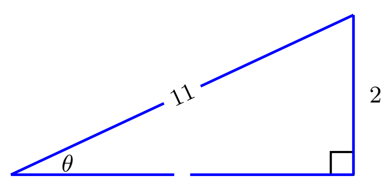
1. Find the angle θ



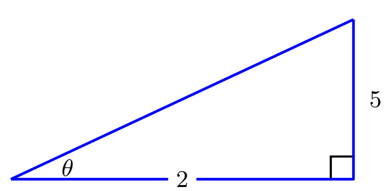
2. Find the angle θ



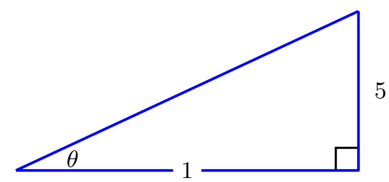
3. Find the angle θ



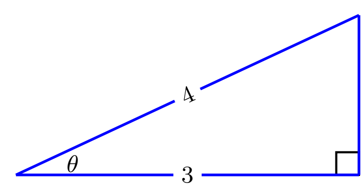
4. Find the angle θ



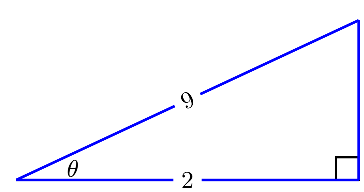
5. Find the angle θ



6. Find the angle θ



7. Find the angle θ



8.

$$\cos^{-1}\left(\cos(30^\circ)\right) = 30^\circ$$

A. TRUE B. FALSE

9.

$$\cos^{-1}\left(\cos(-30^\circ)\right) = -30^\circ$$

A. TRUE B. FALSE

10.

$$\tan^{-1}\left(\tan(30^\circ)\right) = 30^\circ$$

A. TRUE B. FALSE

11.

$$\tan^{-1}\left(\tan(210^\circ)\right) = 210^\circ$$

A. TRUE B. FALSE

12.

$$\tan\left[\tan^{-1}\left(\frac{1}{3}\right)\right] = \frac{1}{3}$$

A. TRUE B. FALSE

13.

$$\tan\left[\tan^{-1}\left(\frac{5}{3}\right)\right] = \frac{5}{3}$$

A. TRUE B. FALSE

14.

$$\cos\left[\cos^{-1}\left(\frac{5}{13}\right)\right] = \frac{5}{13}$$

A. TRUE B. FALSE

15. Compute

$$\sin\left(\cos^{-1}(.3456)\right)$$

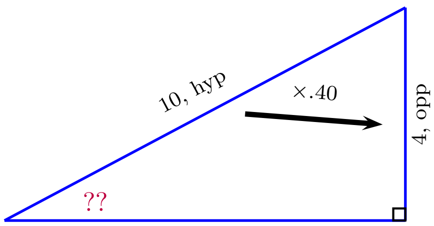
16. Compute

$$\tan\left(\cos^{-1}(.1234)\right)$$

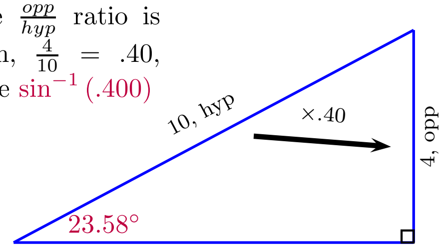
Main Idea:

We have already introduced the idea of inverse functions earlier. While the $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$ functions are great to describe the ratios for a given angle θ, and its respective triangle, the inverse functions $\sin^{-1}(k)$, $\cos^{-1}(k)$ and $\tan^{-1}(k)$ are great for giving the angle when the ratio is known.

Here is an example to help us recall our previous look at the inverse functions:



If the $\frac{opp}{hyp}$ ratio is known, $\frac{4}{10} = .40$, we use $\sin^{-1}(.400)$

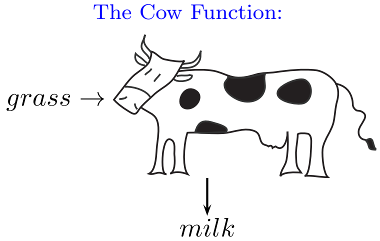


That was an important introduction to the inverse functions. Here we take the opportunity to reconcile how these functions behave once we have extended the trigonometric functions to just about all angles. The complete discussion of these functions requires a reasonable understanding of *ono-to-one* and *onto* functions. While this may not be the appropriate course for such a discussion we will engage in such ideas just enough to gain a broader and adequate perspective of the trigonometric functions.

Inverse Functions:

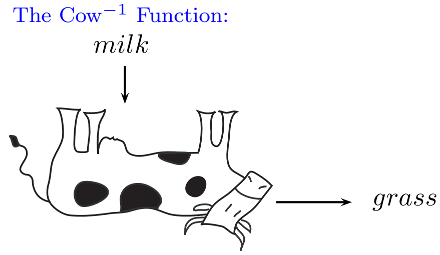
Thus far we have seen the trigonometric functions describe for each angle a particular ratio. In this section we take a moment to contemplate this question but backwards. Suppose we know a particular ratio, can we then find the corresponding angle? The answer is a definitive *yes*. The taylor-made medicine for the job is the study of the inverse functions. The inverse cosine is often called arc-cosine and is usually denoted as $\arccos(z)$ or $\cos^{-1}(z)$. Similarly of arc-sine and arc-tangent. These are defined as you would expect, given a ratio z, $\arccos(z)$ is defined as the angle θ such that $\cos\theta = z$, of course this demands some cautionary and insightful comments, which we will add soon. The following diagram should help solidify the concept of the inverse functions.

The Cow Function:



$\text{cow}(grass)=milk$

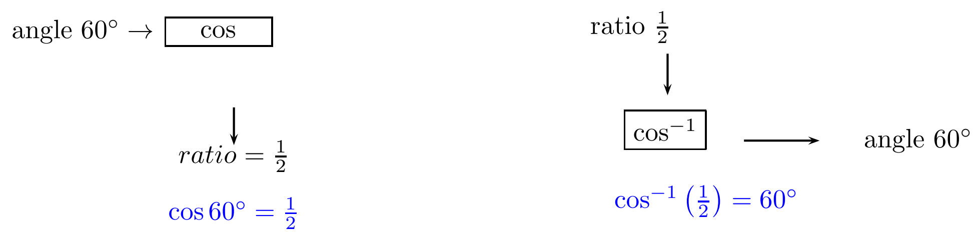
The Cow^{-1} Function:



$\text{cow}^{-1}(milk)=grass$

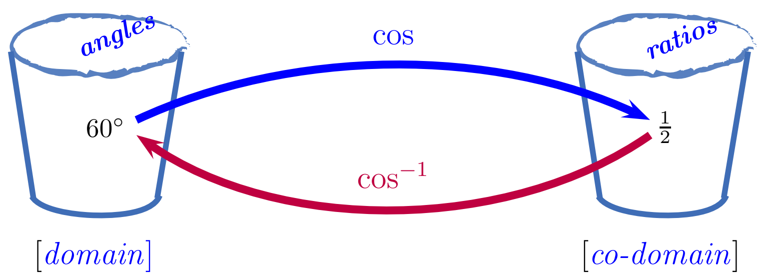
Now, we apply the idea to cosine function at 60°

The Cosine Function: The Cosine Inverse Function:

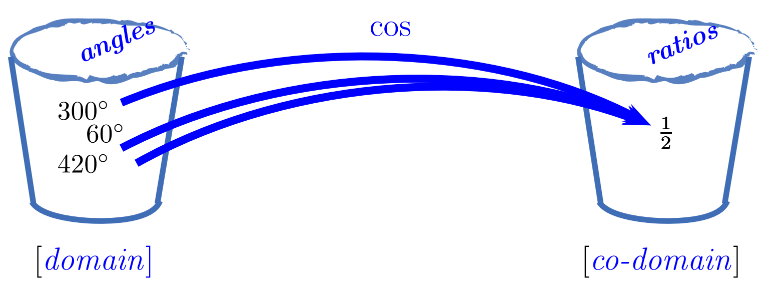


The Inverse Functions:

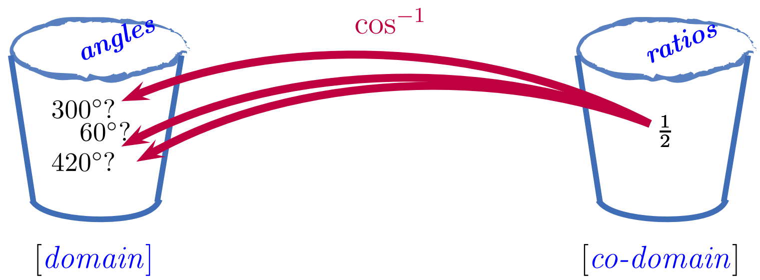
Here is yet another way to illustrate the role of the inverse trigonometric functions. Let us continue with the same example.



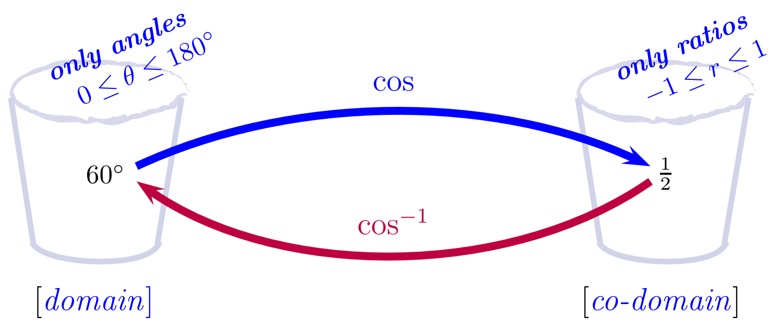
Consider the possibility that feeding the cow "hot-dogs" also produces 'milk'. This adds a small complication when calculating what the inverse function, cow^{-1}, does to 'milk'. Namely, it's tricky to decide wether $cow^{-1} = grass$ or $cow^{-1} = hot - dogs$. In fact, in such a situation the cow function does not have an inverse. In general, when a function is not one-to-one it does not have an inverse. Such is the case for all our trigonometric functions. Although a complete discussion of one-to-one functions may not be appropriate here, we are obliged to at least scratch the surface of the topic, at least enough to understand the basic intricacies of trigonometric inverse functions. With that in mind consider the following angles, all of which result in a cosine value of $\frac{1}{2}$.



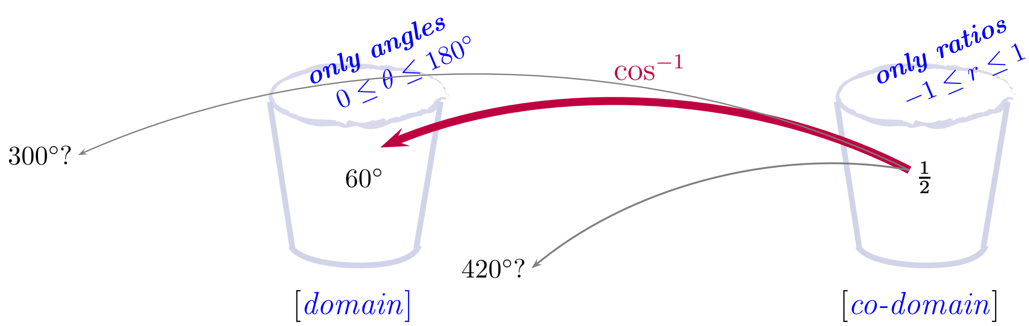
Now, consider the inverse cosine function, that is if the ratio of adjacent to hypothenuse is $\frac{1}{2}$, can we recover the angle? The answer is a resounding *no!*. There are too many [more than one] angles for which such ratio is $\frac{1}{2}$. When this happens we say that the function is not one-to-one.



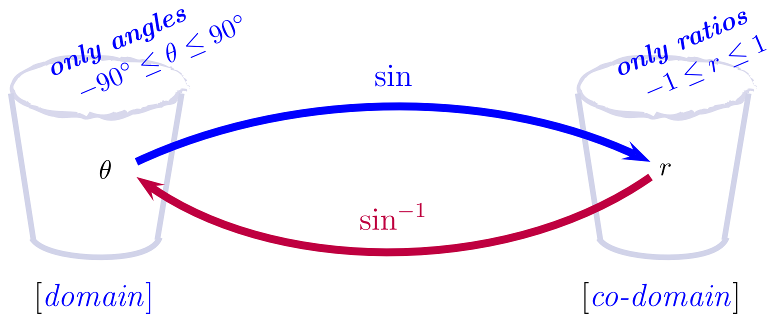
In some way, this is a remediable problem. The popular and almost universally accepted approach is to modify the domain on the cosine function as to limit angles to *only* angles between 0° and 180°. At the same time, the ratios have to be restricted to values between -1 and 1.



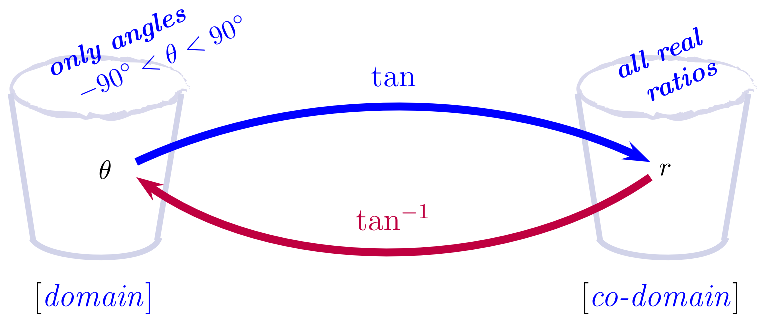
Note how the inverse problem is resolved:



Of the many different angles for which the cosine is 1/2, only one is in the adequate range, thus there is no ambiguity as to which angle the ratio is assigned to. SImilarly, the domain for the sine function, as well as the tangent function can be restricted so that an inverse function can be constructed for each one of these.



For tangent,

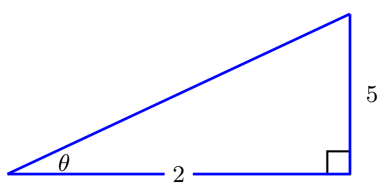


You should know that most scientific calculators are programed to compute inverse trig functions angles according to these ranges. Moreover, you should be aware of alternative names for these functions. $\arccos(x)$ is sometimes used to denote $\cos^{-1}(x)$, and similarly $\arctan(x) = \tan^{-1}(x)$, and $\arcsin(x) = \sin^{-1}(x)$

Examples:

Example:

Find the angle θ



In this case, we know the opposite and the adjacent sides. The function describing such ratio is the tangent, thus

$$\tan\theta = \frac{5}{2}$$

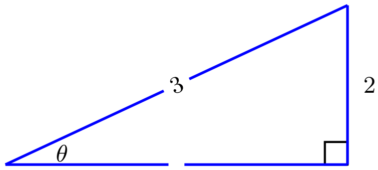
Since this is not a famous ratio, we allow ourselves use of a calculator to estimate the sought angle.

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{5}{2}\right) \\ &\approx 1.19 \qquad \text{(in radians [see calculator mode]... OR...)} \\ &\approx 68.199° \qquad \text{(in degrees [see calculator mode])}\end{aligned}$$

... it should be noted that we will revisit the equation $\tan\theta = \frac{5}{2}$ under a different context, where we will solve it completely, not limited to the domain and codomain of the arctan function.

Example:

Find the angle θ



In this case, we know the opposite and the hypothenuse sides. The function describing such ratio is the sine, thus

$$\sin\theta = \frac{2}{3}$$

Since this is not a famous ratio, we allow ourselves use of a calculator to estimate the sought angle.

$$\theta = \sin^{-1}\left(\frac{2}{3}\right)$$
$$\approx 0.73 \qquad \text{(in radians [see calculator mode]... OR...)}$$
$$\approx 41.81^\circ \qquad \text{(in degrees [see calculator mode])}$$

... it should be noted that we will revisit the equation $\sin\theta = \frac{2}{3}$ under a different context, where we will solve it completely, not limited to the domain and codomain of the arcsin function.