

## Famous IDs: Def. Identities

### Main Idea

Trigonometric identities are a significant and essential portion of any trigonometry introductory course. In the first couple sections, we have introduced what 'identity' means, and we have introduced some of the most common and useful ways to establish whether a proposed equation is or is not an identity, namely, working on each side, looking at the graphs, tweaking a known identity, or something brilliant and creative. Moreover, the last section should have provided ample practice in proving identities. In fact, some of these will translate, almost verbatim, into very famous trig identities.

For the remainder of this chapter, we will turn our attention exclusively to trigonometric identities. Some of these trig identities are incredibly famous [and useful] and others are incredibly not famous. We will prove both types, the famous ones because they are famous, and the non-famous ones just for practice.

To that end, the time is here to introduce some of the most famous trigonometric identities.

(see <http://www.mathhands.com/104/free/ids.pdf>)

### First Paragraph: The Definition Identities

#### Definition Identities

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

It should be noted that the following identities require no proving, since they follow immediately from the definitions of each of the functions.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cot \theta = \frac{\text{adj}}{\text{hyp}}$$

**EXAMPLE (working on one side)** Prove the following is an identity.

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Solution: Since every angle has a reference angle, the angle,  $\theta$ , has a reference triangle. Let us label it as usual with opp, hyp, and adj sides, as appropriate. Then...

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\begin{aligned}
 & \frac{\sin \theta}{\cos \theta} && \text{(right side)} \\
 = & \frac{\text{opp}}{\text{hyp}} && \text{(use definitions)} \\
 = & \frac{\text{opp}}{\text{hyp}} \cdot \frac{\text{hyp}}{\text{adj}} && \text{(algebra)} \\
 = & \frac{\text{opp}}{\text{adj}} && \text{(algebra)} \\
 = & \tan \theta && \text{(def.)}
 \end{aligned}$$

Therefore,

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

The rest of the identities are proven in a similar manner and are left as important exercises for the student. With that, we now turn our attention to the second paragraph on the famous identity sheet.

**Second Paragraph:** The Co-Function Identities

Co-Function Identities

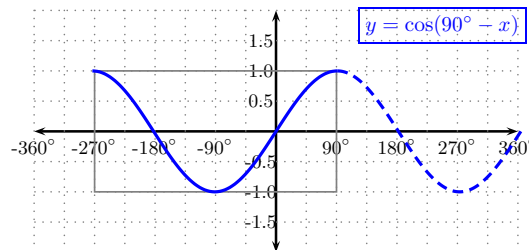
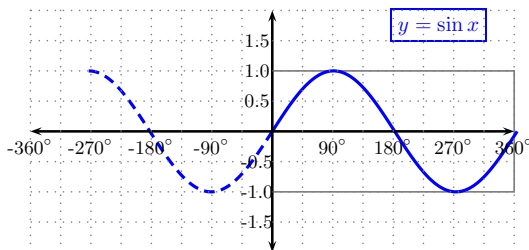
$$\begin{array}{ll}
 \sin \theta = \cos(90^\circ - \theta) & \tan \theta = \cot(90^\circ - \theta) \\
 \sec \theta = \csc(90^\circ - \theta) & \sin \theta = \cos(\theta - 90^\circ) \\
 -\cos \theta = \sin(\theta - 90^\circ) & \cos \theta = \sin(90^\circ - \theta)
 \end{array}$$

**EXAMPLE: (look at graphs)** Prove the following is an identity.

$$\sin x = \cos(90^\circ - x)$$

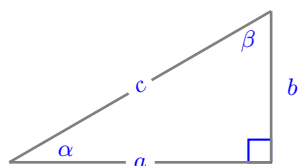
**solution:**

We will look at each of the graphs and compare to see if these are convincingly equal( Note to graph  $\cos(90^\circ - x)$ , we could use the *prep, scale, shift* method, thus we will graph the equivalent [prepared] version  $\cos(-(x - 90^\circ))$ ... ie flipped then shifted right  $90^\circ$ ).

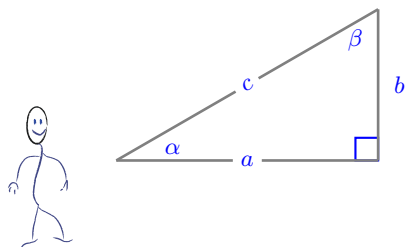


Some problems are so nice that they generate multiply solutions, in multiple ways. Here, we can't help but present a second solution to the exact same question. We will next prove the identity  $\sin x = \cos(90^\circ - x)$  not by looking at the respective graphs but by something a little more interesting.

Suppose the angle  $x$  is between  $0$  and  $90^\circ$ , then the general reference triangle looks as such:



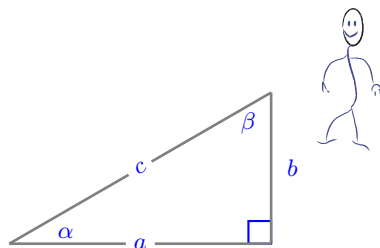
The key is to look at this triangle from different points of view. From the angle  $\alpha$  point of view, the opposite side is  $b$  and the hypotenuse is  $c$ . Thus,



Thus,

$$\sin \alpha = \frac{b}{c}$$

On the other hand, if from the **Complimentary** angle,  $\beta$ , the adjacent side is  $b$ .



Thus,

$$\cos \beta = \frac{b}{c}$$

Therefore,

$$\sin \alpha = \cos \beta$$

moreover,  $\alpha$  and  $\beta$  are complimentary, thus  $\alpha + \beta = 90^\circ$  OR  $\beta = 90^\circ - \alpha$

Therefore:

$$\sin \alpha = \cos (90^\circ - \alpha)$$

The above argument assume  $\alpha$  is between 0 and  $90^\circ$ , the reader is invited to generalize the argument for negative angles or for angles beyond the first quadrant, in fact the identity  $\sin \alpha = \cos (90^\circ - \alpha)$  holds true for any real angle,  $\alpha$ .

Famous IDs: Def. Identities

1. [very famous] Determine if the following is an identity or not: Explain

$$\cot x = \frac{\cos x}{\sin x}$$

2. [very famous] Determine if the following is an identity or not: Explain

$$\sec x = \frac{1}{\cos x}$$

3. [very famous] Determine if the following is an identity or not: Explain

$$\csc x = \frac{1}{\sin x}$$

4. [very famous] Determine if the following is an identity or not: Explain

$$\sin(2x) = 2 \sin x$$

5. [very famous] Determine if the following is an identity or not: Explain

$$\sin x = \frac{1}{\csc x}$$

6. [very famous] Determine if the following is an identity or not: Explain

$$\cos x = \frac{1}{\sec x}$$

7. [little famous] Determine if the following is an identity or not: Explain

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

8. [little famous] Determine if the following is an identity or not: Explain

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$$

9. [little famous] Determine if the following is an identity or not: Explain

$$\sec^2 x = \frac{1}{\cos^2 x}$$

10. [very famous] Determine if the following is an identity or not: Explain

$$\tan \theta = \cot (90^\circ - \theta)$$

11. [very famous] Determine if the following is an identity or not: Explain

$$\sec \theta = \csc (90^\circ - \theta)$$

**Trigonometry**  
**Sec. 03 exercises**

12. [very famous] Determine if the following is an identity or not: Explain

$$\sin \theta = \cos(\theta - 90^\circ)$$

13. [very famous] Determine if the following is an identity or not: Explain

$$-\cos \theta = \sin(\theta - 90^\circ)$$

14. [very famous] Determine if the following is an identity or not: Explain

$$\cos \theta = \sin(90^\circ - \theta)$$