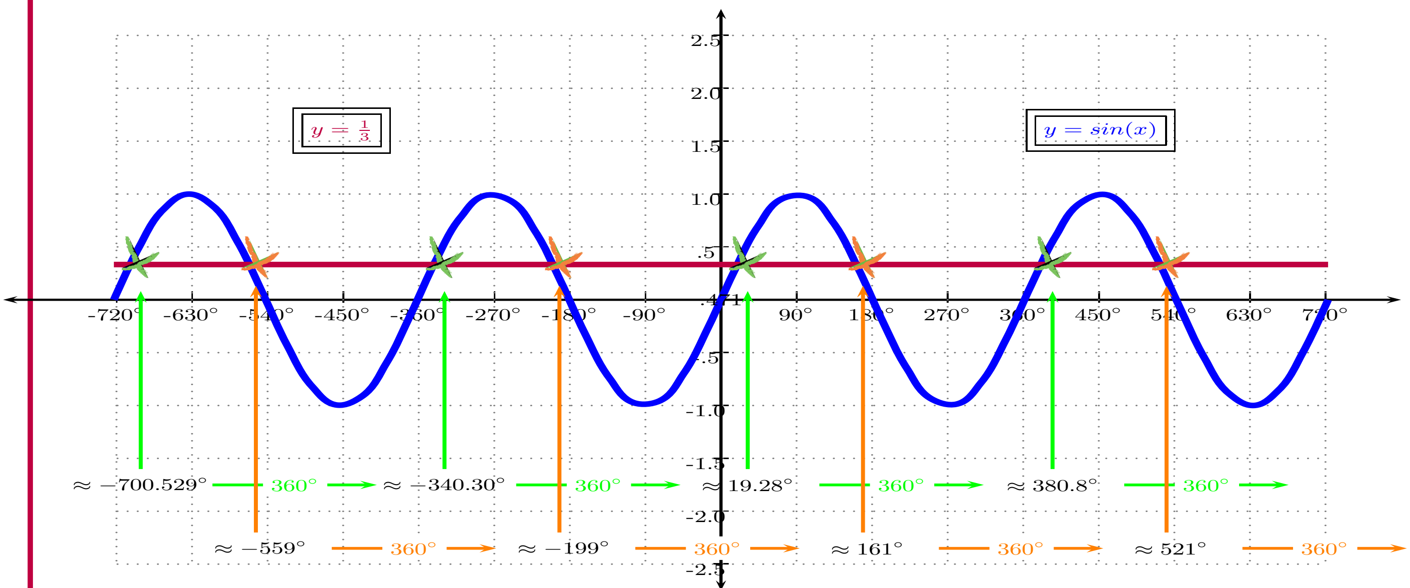
Basic Trig Eqs:

Solve

$$\sin(x) = \frac{1}{3}$$

Solution:
First we graph $y \approx 0.333$ then we graph $y = \sin(x)$, then we mark the intersections. These are the solutions. Clear from the graph is that we have infinite many of them.
Of these, the first one is determined by using a calculator to estimate $\sin^{-1}(0.333) \approx 19.471°$. This solution is the only one provided by the $\sin^{-1}$ function.



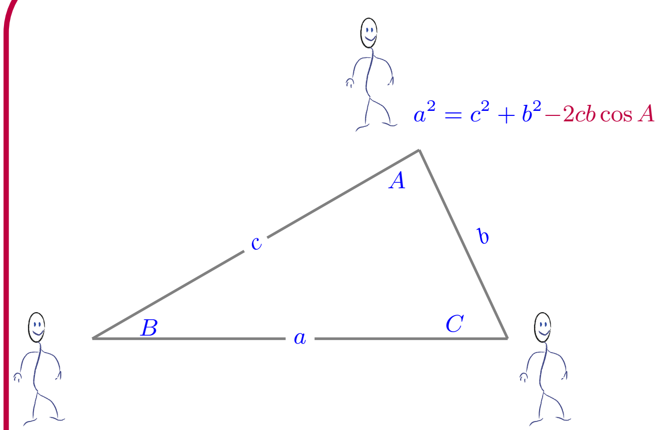
We conclude the solution by describing all possible solutions. We summarize, each solution occurs either in the 'uphill bucket' or in the 'downhill bucket' thus if x is a solution, it must be one of the following values:

$$x_k \approx 19.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 160.529° + k360° \qquad \text{for } k \in \mathbb{Z}$$

The LAW of CoSINES

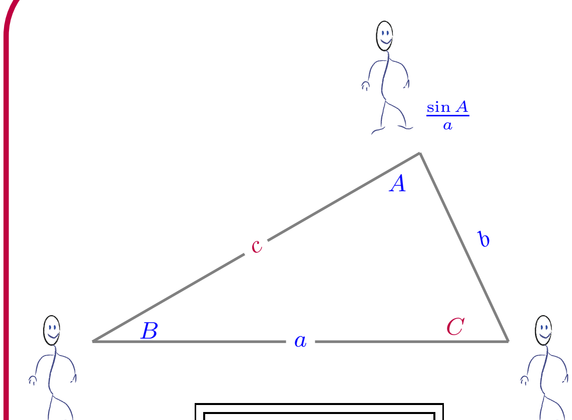


$a^2 = c^2 + b^2 - 2cb\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

The LAW of SINES

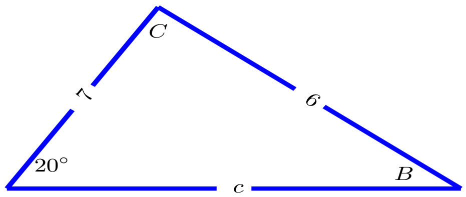


$\frac{\sin A}{a}$ $\frac{\sin B}{b}$ $\frac{\sin C}{c}$

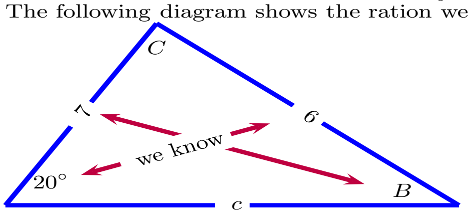
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Solving Triangles:

Solve



We first note that we know the ratio $\frac{\sin 20^\circ}{6}$, and we know the side $b = 7$. Thus.. we can apply the law of sines to solve for the angle B. The following diagram shows the ration we know and the ratio we will compare it to.



$$\frac{\sin 20^\circ}{6} = \frac{\sin B}{7} \qquad \text{(applying the law of sines)}$$

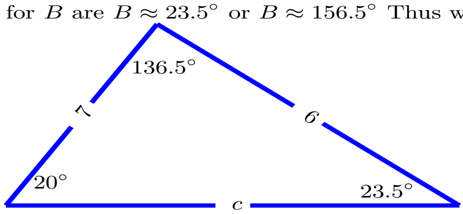
$$7 \cdot \frac{\sin 20^\circ}{6} = \sin B \qquad \text{(algebra)}$$

$$\sin B = 7 \cdot \frac{\sin 20^\circ}{6} \qquad \text{(now, we will use calc. to approximate)}$$

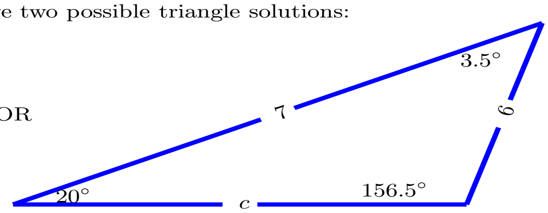
$$\sin B \approx 0.399 \qquad \text{(now we need to solve correctly and completely)}$$

$$B \approx \ldots, -203.5^\circ, 23.5^\circ, 156.5^\circ, 383.5^\circ, \ldots$$

This is as far as the law of sines can help us.. but we can take it from here. Assuming B is an interior angle of triangle [in flatland.. Euclidean space] we can assume it must measure somewhere between 0 and 180°. The only such choices in the above list of candidates for B are $B \approx 23.5^\circ$ or $B \approx 156.5^\circ$ Thus we have two possible triangle solutions:



OR

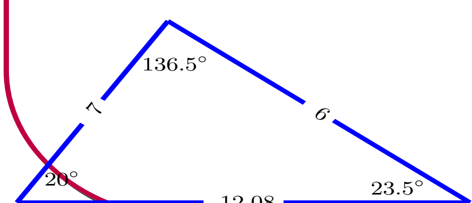


then we just solve for c in each case...
For the first triangle:

$$\frac{c}{\sin 136.5^\circ} = \frac{6}{\sin 20^\circ}$$
$$c = \frac{6}{\sin 20^\circ} \cdot \sin 136.5^\circ$$
$$c \approx 12.08$$

or

$$\frac{c}{\sin 3.5^\circ} = \frac{6}{\sin 20^\circ}$$
$$c = \frac{6}{\sin 20^\circ} \cdot \sin 3.5^\circ$$
$$c \approx 1.07$$



OR