Solving Trig Equations: The Almost-Easy Ones

Main Idea

In the last section we solved equations such as:

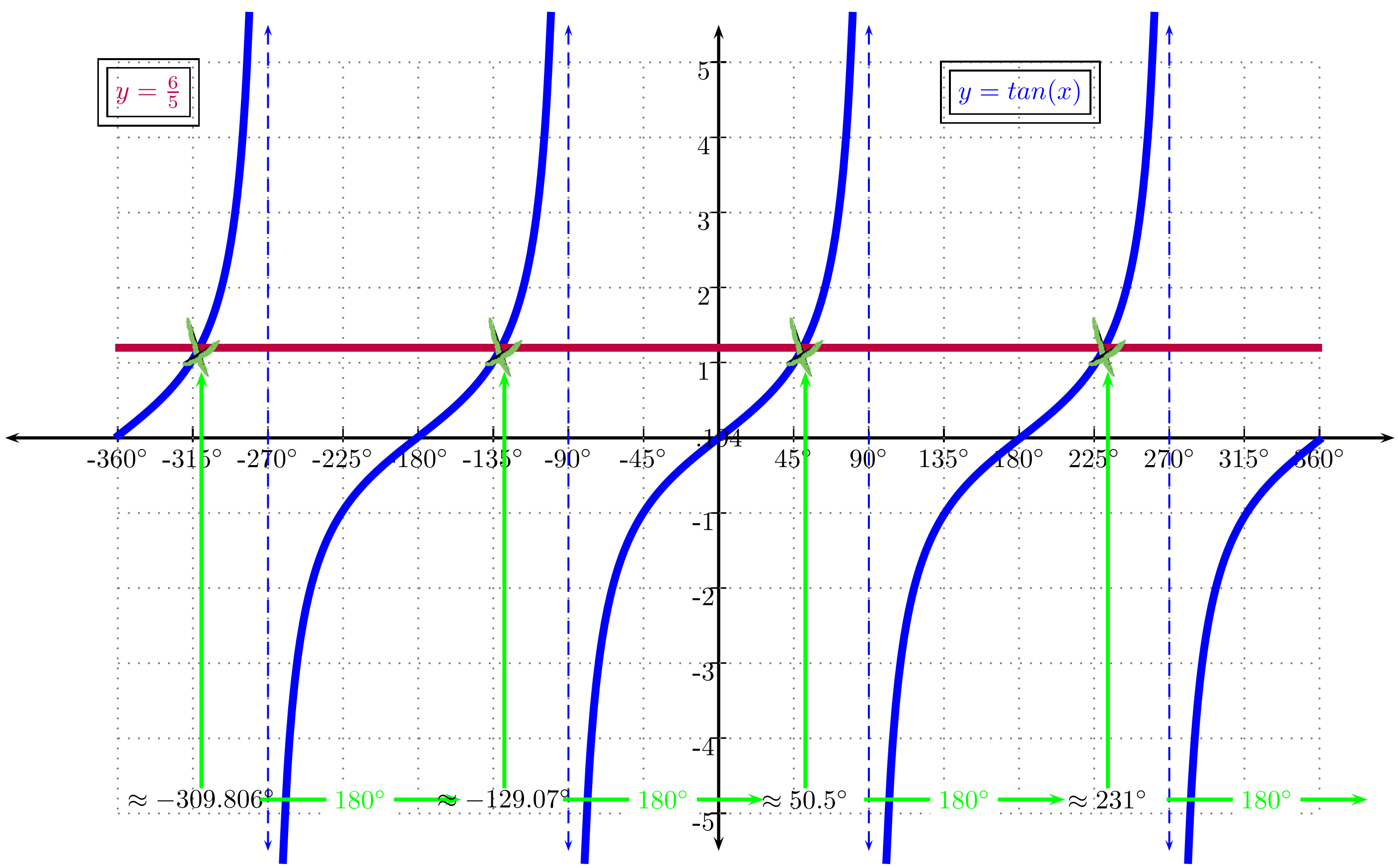
Solve

$$tan\,(x) = \frac{6}{5}$$

Solution:

First we graph $y \approx 1.2$ then we graph $y = tan(x)$, then we mark the intersections. These are the solutions. Clear from the graph is that we have infinite many of them.

Of these, the first one is determined by using a calculator to estimate $tan^{-1}(1.2) \approx 50.194°$. This solution is the only one provided by the tan^{-1} function.



We conclude the solution by describing all possible values of x.

$$x \approx 50.194° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Now consider if in the equation

$$\tan(x) = \frac{6}{5}$$

the x was replaced with something else, such as θ. One could solve it the same manner, with the same results:

Example:
Solve

$$tan\,(\theta) = \frac{6}{5}$$

Solution:
The solution for possible values of θ:

$$\theta \approx 50.194° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Example:
Solve

$$tan\,(blah) = \frac{6}{5}$$

Solution:
The solution for possible values of $blah$:

$$blah \approx 50.194° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Example:
Solve

$tan\,($  $) = \frac{6}{5}$

Solution:
The solution for possible values of :

 $\approx 50.194° + k180° \qquad \text{for } k \in \mathbb{Z}$

Now the punch-line...

Example:
Solve

$$tan\,(2x + 30°) = \frac{6}{5}$$

Solution:
From solving the easy version of the equation we obtain

$$2x + 30° \approx 50.194° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Therefore:

$$\begin{aligned} 2x + 30° &\approx 50.194° + k180° \\ 2x &\approx 20.194° + k180° \\ x &\approx 10.1° + k90° \end{aligned}$$

Example:

Solve

$$tan\left(3x + 45°\right) = -\frac{4}{5}$$

Solution:

From solving the easy version of the equation we obtain

$$3x + 45° \approx -38.66° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Therefore:

$$\begin{aligned} 3x + 45° &\approx -38.66° + k180° \\ 3x &\approx -83.66° + k180° \\ x &\approx -27.89° + k60° \end{aligned}$$

Example:

Solve

$$tan\left(2x - 45°\right) = \frac{7}{3}$$

Solution:

From solving the easy version of the equation we obtain

$$2x - 45° \approx 66.801° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Therefore:

$$\begin{aligned} 2x - 45° &\approx 66.801° + k180° \\ 2x &\approx 111.801° + k180° \\ x &\approx 55.9° + k90° \end{aligned}$$

Solving Trig Equations: The Almost-Easy Ones

1. Solve

$$tan\left(2x - 45°\right) = \frac{7}{3}$$

Solution:
From solving the easy version of the equation we obtain

$$2x - 45° \approx 66.801° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Therefore:

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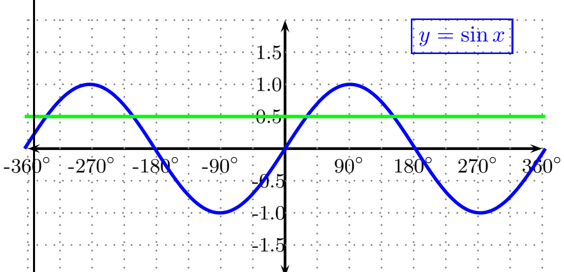
2. Solve the equation

$$\sin x = \frac{1}{2}$$

Solution: Solution: We first draw the $y = \sin x$ graph, then the line $y = \frac{1}{2}$. We then point to the solutions and list them.



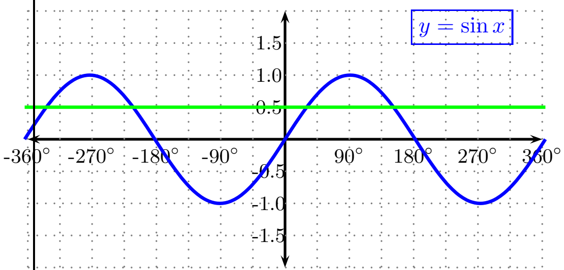
solutions

uphill solutions	$x = 30° + k360°$
downhill solutions	$x = 150° + k360°$

3. Solve the equation

$$\sin(2t - 50°) = \frac{1}{2}$$

Solution: Solution: The key here is to note that this is almost like the previous example. The only difference is x, instead of x, we have the quantity $2t - 50°$. Thus, the answer will be almost like the previous answer except instead of x we will have the quantity $2t - 50°$.



solutions

uphill solutions		$x = 30° + k360°$
	substitute	instead of x, we have $2t - 50°$
		$2t - 50° = 30° + k360°$
		$2t = 80° + k360°$
		$t = 40° + k180°$
downhill solutions		$x = 150° + k360°$
	substitute	instead of x, we have $2t - 50°$
		$2t - 50° = 150° + k360°$
		$2t = 200° + k360°$
		$t = 100° + k180°$

4. Find all solutions

$$\sin(2\theta) = \frac{-1}{2}$$

Solution: FIRST we solve...

The set of all real solutions to $\sin 2\theta = -.5$ is of the form...

$$2\theta = -30.0° + 360°k \qquad \text{or} \qquad 2\theta = -150.0° + 360°k$$

said differently...

$$2\theta = \ldots, -510.0°, -30.0°, -150.0°, 330.0°, \ldots$$

THEN we solve for θ by dividing everything by 2... thus.. .

$$\theta = -15° + 180°k$$

OR

$$\theta = -75° + 180°k$$

5. Find all solutions

$$\sin(3\theta + 90°) = 0$$

Solution: FIRST we solve...

The set of all real solutions to $\sin(3\theta + 90°) = 0$ is of the form...

$$(3\theta + 90°) = 180°k$$

said differently...

$$(3\theta + 90°) = \ldots, -180°, 0°, 180°, 360°, \ldots$$

THEN we solve for θ by subtracting 90° then dividing by 3..... thus.. .

$$\theta = -30° + 60°k$$

6. Find all solutions

$$\sin(2x - 40°) = \frac{1}{3}$$

7. Find all solutions

$$\cos(5x + \pi) = \frac{-1}{2}$$

Solution: FIRST we solve...

The set of all real solutions to $\cos(5x + 180°) = -.5$ is of the form...

$$(5x + 180°) = 120.0° + k360° \qquad \text{or} \qquad (5x + 180°) = 240.0° + k360°$$

said differently...

$$(5x + 180°) = \ldots, -120.0°, 120.0°, 240.0°, 480.0°, \ldots$$

THEN we solve for x by subtracting 180° then dividing by 5..... thus.. .

$$x = \frac{-60° + 360°k}{5}$$

OR

$$x = \frac{60° + 360°k}{5}$$

8. Find all solutions

$$\cos(2t - \pi) = \frac{-1}{3}$$

9. Find all solutions

$$\csc\left(\frac{2x + \pi}{3}\right) = -2$$