Solving Trig Equations: The Almost-Easy Ones

Main Idea

In the last section we solved equations such as:

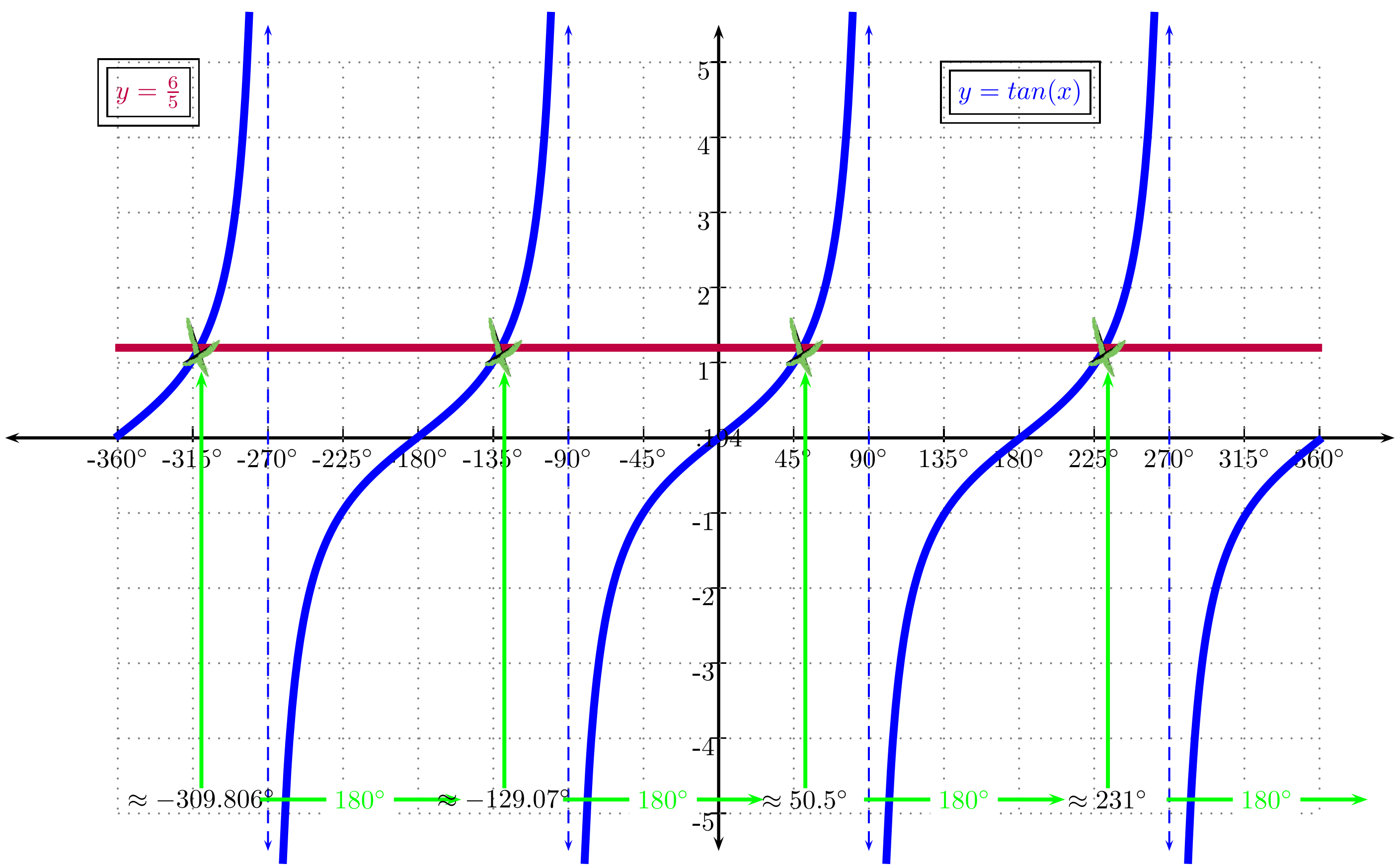
Solve

$$tan\,(x) = \frac{6}{5}$$

Solution:

First we graph $y = 1.2$ then we graph $y = tan(x)$, then we mark the intersections. These are the solutions. Clear from the graph is that we have infinite many of them.

Of these, the first one is determined by using a calculator to estimate $tan^{-1}(1.2) \approx 50.194°$. This solution is the only one provided by the tan^{-1} function.



We conclude the solution by describing all possible values of x.

$$x \approx 50.194° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Now consider if in the equation

$$\tan(x) = \frac{6}{5}$$

the x was replaced with something else, such as θ. One could solve it the same manner, with the same results:

Example:

Solve

$$tan\left(\theta\right) = \frac{6}{5}$$

Solution:

The solution for possible values of θ:

$$\theta \approx 50.194^\circ + k180^\circ \qquad \text{for } k \in \mathbb{Z}$$

Example:

Solve

$$tan\left(blah\right) = \frac{6}{5}$$

Solution:

The solution for possible values of $blah$:

$$blah \approx 50.194^\circ + k180^\circ \qquad \text{for } k \in \mathbb{Z}$$

Example:

Solve

$tan\left(\right.$  $\left.\right) = \frac{6}{5}$

Solution:

The solution for possible values of :

 $\approx 50.194^\circ + k180^\circ \qquad \text{for } k \in \mathbb{Z}$

Now the punch-line...

Example:

Solve

$$tan\left(2x + 30^\circ\right) = \frac{6}{5}$$

Solution:

From solving the easy version of the equation we obtain

$$2x + 30^\circ \approx 50.194^\circ + k180^\circ \qquad \text{for } k \in \mathbb{Z}$$

Therefore:

$$\begin{aligned} 2x + 30^\circ &\approx 50.194^\circ + k180^\circ \\ 2x &\approx 20.194^\circ + k180^\circ \\ x &\approx 20.194^\circ + k90 \end{aligned}$$

Example:

Solve

$$tan\left(3x+45°\right) = -\frac{4}{5}$$

Solution:

From solving the easy version of the equation we obtain

$$3x+45° \approx -38.66° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Therefore:

$$\begin{aligned} 3x+45° &\approx -38.66° + k180° \\ 3x &\approx -83.66° + k180° \\ x &\approx -83.66° + k60 \end{aligned}$$

Example:

Solve

$$tan\left(2x-45°\right) = \frac{7}{3}$$

Solution:

From solving the easy version of the equation we obtain

$$2x-45° \approx 66.801° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Therefore:

$$\begin{aligned} 2x-45° &\approx 66.801° + k180° \\ 2x &\approx 111.801° + k180° \\ x &\approx 111.801° + k90 \end{aligned}$$

Solving Trig Equations: The Almost-Easy Ones

1. Solve
$$tan\left(2x - 45^\circ\right) = \frac{7}{3}$$

2. Solve the equation
$$\sin x = \frac{1}{2}$$

3. Solve the equation
$$\sin(2t - 50^\circ) = \frac{1}{2}$$

4. Find all solutions
$$\sin(2\theta) = \frac{-1}{2}$$

5. Find all solutions
$$\sin(3\theta + 90^\circ) = 0$$

6. Find all solutions
$$\sin(2x - 40^\circ) = \frac{1}{3}$$

7. Find all solutions
$$\cos(5x + \pi) = \frac{-1}{2}$$

8. Find all solutions
$$\cos(2t - \pi) = \frac{-1}{3}$$

9. Find all solutions
$$\csc\left(\frac{2x + \pi}{3}\right) = -2$$