Solving Trig Equations: The Others

Main Idea

In the last sections, we learned to solve very basic trig equations as well as slight variations of these basic ones. In this section, we show techniques which can be used to turn other equations into basic ones. The use of identities will be essential. The general strategy is to get all expressions on one side of the equation and 0 on the other side. Then we try to factor the expressions into simpler terms so that we may use the Zero Factor Theorem. This is, generally speaking, easier said than done. Yet for our class, in may be appropriate so propose mostly equations which do in fact factor in a relatively easy way, so long as we have full command and use of the famous identities.

The following suggestions may often prove helpful.

Solving Trig Eqs: Some Strategies

- Tweak to turn into ez ones.
- Turn everything into sines or cosines.
- Reduce angles so they are all equal.
- Use the famous identities
- Use conjugates
- See if you can factor as a quadratic equation
- Careful not to mult/or divide by zero
- Check answers when finished.

Example:

Solve

$$24sin^2(x) + -38sin(x) + 15 = 0$$

$$24sin^2(x) + -38sin(x) + 15 = 0 \quad \text{(given)}$$

$$\left[4sin(x) + -3\right] \cdot \left[6sin(x) + -5\right] = 0 \quad \text{(factor)}$$

$$4sin(x) + -3 = 0 \quad \text{OR} \quad 6sin(x) + -5 = 0 \quad \text{(Zero Fact Thm)}$$

$$sin(x) = \frac{3}{4} \quad \text{OR} \quad sin(x) = \frac{5}{6} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = \frac{3}{4}$$

Solution:

$$x_k \approx 48.59° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 131.41° + k360° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = \frac{5}{6}$$

Solution:

$$x_k \approx 56.443° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 123.557° + k360° \qquad \text{for } k \in \mathbb{Z}$$

Example:

Solve

$$6cos^2(x) + -13cos(x) + 5 = 0$$

$$6cos^2(x) + -13cos(x) + 5 = 0 \quad \text{(given)}$$
$$\left[2cos(x) + -1\right] \cdot \left[3cos(x) + -5\right] = 0 \quad \text{(factor)}$$
$$2cos(x) + -1 = 0 \quad \text{OR} \quad 3cos(x) + -5 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = \frac{1}{2} \quad \text{OR} \quad cos(x) = \frac{5}{3} \quad \text{(algebra)}$$

Solve

$$cos\,(x) = \frac{1}{2}$$

Solution:

$$x_k \approx 60° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 300° + k360° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$cos\,(x) = \frac{5}{3}$$

no real solution for x

Example:

Solve

$$2sin^2(x) + -5sin(x) + -3 = 0$$

$$2sin^2(x) + -5sin(x) + -3 = 0 \quad \text{(given)}$$
$$\left[2sin(x) + 1\right] \cdot \left[1sin(x) + -3\right] = 0 \quad \text{(factor)}$$
$$2sin(x) + 1 = 0 \quad \text{OR} \quad 1sin(x) + -3 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = -\frac{1}{2} \quad \text{OR} \quad sin(x) = 3 \quad \text{(algebra)}$$

Solve

$$sin\,(x) = -\frac{1}{2}$$

Solution:

$$x_k \approx -30° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 210° + k360° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = 3$$

no real solution for x

Solving Trig Equations: The Others

1. Solve

$$3sin^2(x) + sin(x) = 0$$

Solution:

$$3sin^2(x) + 1sin(x) = 0 \quad \text{(given)}$$
$$sin(x)\left[3sin(x) + 1\right] = 0 \quad \text{(factor)}$$
$$sin(x) = 0 \quad \text{OR} \quad 3sin(x) + 1 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = 0 \quad \text{OR} \quad sin(x) = -\frac{1}{3} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = 0$$

Solution:

$$x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = -\frac{1}{3}$$

Solution:

$$x_k \approx -19.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 199.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

2. Solve

$$2sin^2(x) + 5sin(x) = 0$$

Solution:

$$2sin^2(x) + 5sin(x) = 0 \quad \text{(given)}$$
$$sin(x)\left[2sin(x) + 5\right] = 0 \quad \text{(factor)}$$
$$sin(x) = 0 \quad \text{OR} \quad 2sin(x) + 5 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = 0 \quad \text{OR} \quad sin(x) = -\frac{5}{2} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = 0$$

Solution:

$$x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = -\frac{5}{2}$$

no real solution for x

3. Solve

$$5cos^2(x) + cos(x) = 0$$

Solution:

$$5cos^2(x) + 1cos(x) = 0 \quad \text{(given)}$$
$$cos(x)\left[5cos(x) + 1\right] = 0 \quad \text{(factor)}$$
$$cos(x) = 0 \quad \text{OR} \quad 5cos(x) + 1 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = 0 \quad \text{OR} \quad cos(x) = -\frac{1}{5} \quad \text{(algebra)}$$

Solve
$$cos\,(x) = 0$$

Solution:

$$x_k = 90° + k180° \quad \text{for } k \in \mathbb{Z}$$

Solve
$$cos\,(x) = -\frac{1}{5}$$

Solution:

$$x_k \approx 101.537° + k360° \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 258.463° + k360° \quad \text{for } k \in \mathbb{Z}$$

4. Solve
$$6cos^2(x) + 2cos(x) = 0$$

Solution:

$$6cos^2(x) + 2cos(x) = 0 \quad \text{(given)}$$
$$cos(x)\left[6cos(x) + 2\right] = 0 \quad \text{(factor)}$$
$$cos(x) = 0 \quad \text{OR} \quad 6cos(x) + 2 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = 0 \quad \text{OR} \quad cos(x) = -\frac{1}{3} \quad \text{(algebra)}$$

Solve
$$cos\,(x) = 0$$

Solution:

$$x_k = 90° + k180° \quad \text{for } k \in \mathbb{Z}$$

Solve
$$cos\,(x) = -\frac{1}{3}$$

Solution:

$$x_k \approx 109.471° + k360° \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 250.529° + k360° \quad \text{for } k \in \mathbb{Z}$$

5. Solve
$$8sin^2(x) + -2sin(x) + -3 = 0$$

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Solution:

$$8sin^2(x) + -2sin(x) + -3 = 0 \quad \text{(given)}$$
$$\left[2sin(x)+1\right]\cdot\left[4sin(x) + -3\right] = 0 \quad \text{(factor)}$$
$$2sin(x)+1=0 \quad \text{OR} \quad 4sin(x) + -3 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = -\frac{1}{2} \quad \text{OR} \quad sin(x) = \frac{3}{4} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = -\frac{1}{2}$$

Solution:

$$x_k \approx -30^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 210^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = \frac{3}{4}$$

Solution:

$$x_k \approx 48.59^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 131.41^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

6. Solve

$$4sin^2(x) + 0sin(x) + -1 = 0$$

Solution:

$$4sin^2(x) + 0sin(x) + -1 = 0 \quad \text{(given)}$$
$$\left[2sin(x)+1\right]\cdot\left[2sin(x) + -1\right] = 0 \quad \text{(factor)}$$
$$2sin(x)+1=0 \quad \text{OR} \quad 2sin(x) + -1 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = -\frac{1}{2} \quad \text{OR} \quad sin(x) = \frac{1}{2} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = -\frac{1}{2}$$

Solution:

$$x_k \approx -30^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 210^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = \frac{1}{2}$$

Solution:

$$x_k \approx 30^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 150^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

7. Solve

$$4cos^2(x) + 0cos(x) + -1 = 0$$

Solution:

$$4cos^2(x) + 0cos(x) + -1 = 0 \quad \text{(given)}$$
$$\big[2cos(x) + 1\big] \cdot \big[2cos(x) + -1\big] = 0 \quad \text{(factor)}$$
$$2cos(x) + 1 = 0 \quad \text{OR} \quad 2cos(x) + -1 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = -\frac{1}{2} \quad \text{OR} \quad cos(x) = \frac{1}{2} \quad \text{(algebra)}$$

Solve
$$cos\,(x) = -\frac{1}{2}$$

Solution:

$$x_k \approx 120^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$
OR
$$x_k \approx 240^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

Solve
$$cos\,(x) = \frac{1}{2}$$

Solution:

$$x_k \approx 60^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$
OR
$$x_k \approx 300^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

8. Solve
$$6cos^2(x) + -1cos(x) + -1 = 0$$

Solution:

$$6cos^2(x) + -1cos(x) + -1 = 0 \quad \text{(given)}$$
$$\big[3cos(x) + 1\big] \cdot \big[2cos(x) + -1\big] = 0 \quad \text{(factor)}$$
$$3cos(x) + 1 = 0 \quad \text{OR} \quad 2cos(x) + -1 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = -\frac{1}{3} \quad \text{OR} \quad cos(x) = \frac{1}{2} \quad \text{(algebra)}$$

Solve
$$cos\,(x) = -\frac{1}{3}$$

Solution:

$$x_k \approx 109.471^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$
OR
$$x_k \approx 250.529^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

Solve
$$cos\,(x) = \frac{1}{2}$$

Solution:

$$x_k \approx 60^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$
OR
$$x_k \approx 300^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

9. Solve
$$12sin^2(x) + -5sin(x) + -3 = 0$$

Solution:

$$12sin^2(x) + -5sin(x) + -3 = 0 \quad \text{(given)}$$
$$\left[3sin(x) + 1\right] \cdot \left[4sin(x) + -3\right] = 0 \quad \text{(factor)}$$
$$3sin(x) + 1 = 0 \quad \text{OR} \quad 4sin(x) + -3 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = -\frac{1}{3} \quad \text{OR} \quad sin(x) = \frac{3}{4} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = -\frac{1}{3}$$

Solution:

$$x_k \approx -19.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 199.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = \frac{3}{4}$$

Solution:

$$x_k \approx 48.59° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 131.41° + k360° \qquad \text{for } k \in \mathbb{Z}$$

10. Solve

$$4sin^2(x) + 8sin(x) + -5 = 0$$

Solution:

$$4sin^2(x) + 8sin(x) + -5 = 0 \quad \text{(given)}$$
$$\left[2sin(x) + 5\right] \cdot \left[2sin(x) + -1\right] = 0 \quad \text{(factor)}$$
$$2sin(x) + 5 = 0 \quad \text{OR} \quad 2sin(x) + -1 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = -\frac{5}{2} \quad \text{OR} \quad sin(x) = \frac{1}{2} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = -\frac{5}{2}$$

no real solution for x

Solve

$$sin\,(x) = \frac{1}{2}$$

Solution:

$$x_k \approx 30° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 150° + k360° \qquad \text{for } k \in \mathbb{Z}$$

11. Solve

$$10cos^2(x) + -18cos(x) + -4 = 0$$

Solution:

$$10cos^2(x) + -18cos(x) + -4 = 0 \quad \text{(given)}$$
$$\left[5cos(x) + 1\right] \cdot \left[2cos(x) + -4\right] = 0 \quad \text{(factor)}$$
$$5cos(x) + 1 = 0 \quad \text{OR} \quad 2cos(x) + -4 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = -\frac{1}{5} \quad \text{OR} \quad cos(x) = 2 \quad \text{(algebra)}$$

Solve

$$cos\,(x) = -\frac{1}{5}$$

Solution:

$$x_k \approx 101.537° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 258.463° + k360° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$cos\,(x) = 2$$

no real solution for x

12. Solve

$$60cos^2(x) + 2cos(x) + -6 = 0$$

Solution:

$$60cos^2(x) + 2cos(x) + -6 = 0 \quad \text{(given)}$$
$$\left[6cos(x) + 2\right] \cdot \left[10cos(x) + -3\right] = 0 \quad \text{(factor)}$$
$$6cos(x) + 2 = 0 \quad \text{OR} \quad 10cos(x) + -3 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = -\frac{1}{3} \quad \text{OR} \quad cos(x) = \frac{3}{10} \quad \text{(algebra)}$$

Solve

$$cos\,(x) = -\frac{1}{3}$$

Solution:

$$x_k \approx 109.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 250.529° + k360° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$cos\,(x) = \frac{3}{10}$$

Solution:

$$x_k \approx 72.542° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 287.458° + k360° \qquad \text{for } k \in \mathbb{Z}$$

13. Solve

$$3 + sin(x) = 3cos^2(x)$$

Solution:

$$3 + sin(x) = 3cos^2(x) \quad \text{(given)}$$
$$3 + sin(x) = 3(1 - sin^2(x)) \quad \text{(famous Pyth identity)}$$
$$3 + sin(x) = 3 - 3sin^2(x) \quad \text{(famous Pyth identity)}$$
$$3sin^2(x) + 1sin(x) = 0 \quad \text{(given)}$$
$$sin(x)\left[3sin(x) + 1\right] = 0 \quad \text{(factor)}$$
$$sin(x) = 0 \quad \text{OR} \quad 3sin(x) + 1 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = 0 \quad \text{OR} \quad sin(x) = -\frac{1}{3} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = 0$$

Solution:

$$x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = -\frac{1}{3}$$

Solution:

$$x_k \approx -19.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 199.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

14. Solve

$$3 + -cos(x) = 3sin^2(x)$$

Solution:

$$3 + -cos(x) = 3sin^2(x) \quad \text{(given)}$$
$$3 + -cos(x) = 3(1 - cos^2(x)) \quad \text{(famous Pyth identity)}$$
$$3 + -cos(x) = 3 - 3cos^2(x) \quad \text{(famous Pyth identity)}$$
$$3cos^2(x) + -1cos(x) = 0 \quad \text{(given)}$$
$$cos(x)\left[3cos(x) + -1\right] = 0 \quad \text{(factor)}$$
$$cos(x) = 0 \quad \text{OR} \quad 3cos(x) + -1 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = 0 \quad \text{OR} \quad cos(x) = \frac{1}{3} \quad \text{(algebra)}$$

Solve

$$cos\,(x) = 0$$

Solution:

$$x_k = 90° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$cos\,(x) = \frac{1}{3}$$

Solution:

$$x_k \approx 70.529° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 289.471° + k360° \qquad \text{for } k \in \mathbb{Z}$$

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15. Solve

$$2 + sin(x) = 2cos^2(x)$$

Solution:

$$2 + sin(x) = 2cos^2(x) \quad \text{(given)}$$
$$2 + sin(x) = 2\left(1 - sin^2(x)\right) \quad \text{(famous Pyth identity)}$$
$$2 + sin(x) = 2 - 2sin^2(x) \quad \text{(famous Pyth identity)}$$
$$2sin^2(x) + 1sin(x) = 0 \quad \text{(given)}$$
$$sin(x)\left[2sin(x) + 1\right] = 0 \quad \text{(factor)}$$
$$sin(x) = 0 \quad \text{OR} \quad 2sin(x) + 1 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = 0 \quad \text{OR} \quad sin(x) = -\frac{1}{2} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = 0$$

Solution:

$$x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = -\frac{1}{2}$$

Solution:

$$x_k \approx -30° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 210° + k360° \qquad \text{for } k \in \mathbb{Z}$$

16. Solve

$$2 + -3sin(x) = 2cos^2(x)$$

Solution:

$$2 + -3sin(x) = 2cos^2(x) \quad \text{(given)}$$
$$2 + -3sin(x) = 2\left(1 - sin^2(x)\right) \quad \text{(famous Pyth identity)}$$
$$2 + -3sin(x) = 2 - 2sin^2(x) \quad \text{(famous Pyth identity)}$$
$$2sin^2(x) + -3sin(x) = 0 \quad \text{(given)}$$
$$sin(x)\left[2sin(x) + -3\right] = 0 \quad \text{(factor)}$$
$$sin(x) = 0 \quad \text{OR} \quad 2sin(x) + -3 = 0 \quad \text{(Zero Fact Thm)}$$
$$sin(x) = 0 \quad \text{OR} \quad sin(x) = \frac{3}{2} \quad \text{(algebra)}$$

Solve

$$sin\,(x) = 0$$

Solution:

$$x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$sin\,(x) = \frac{3}{2}$$

no real solution for x

17. Solve

$$4 + -3cos(x) = 4sin^2(x)$$

Solution:

$$4 + -3cos(x) = 4sin^2(x) \quad \text{(given)}$$
$$4 + -3cos(x) = 4\left(1 - cos^2(x)\right) \quad \text{(famous Pyth identity)}$$
$$4 + -3cos(x) = 4 - 4cos^2(x) \quad \text{(famous Pyth identity)}$$
$$4cos^2(x) + -3cos(x) = 0 \quad \text{(given)}$$
$$cos(x)\left[4cos(x) + -3\right] = 0 \quad \text{(factor)}$$
$$cos(x) = 0 \quad \text{OR} \quad 4cos(x) + -3 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = 0 \quad \text{OR} \quad cos(x) = \frac{3}{4} \quad \text{(algebra)}$$

Solve

$$cos\,(x) = 0$$

Solution:

$$x_k = 90° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$cos\,(x) = \frac{3}{4}$$

Solution:

$$x_k \approx 41.41° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 318.59° + k360° \qquad \text{for } k \in \mathbb{Z}$$

18. Solve

$$6 + -5cos(x) = 6sin^2(x)$$

Solution:

$$6 + -5cos(x) = 6sin^2(x) \quad \text{(given)}$$
$$6 + -5cos(x) = 6\left(1 - cos^2(x)\right) \quad \text{(famous Pyth identity)}$$
$$6 + -5cos(x) = 6 - 6cos^2(x) \quad \text{(famous Pyth identity)}$$
$$6cos^2(x) + -5cos(x) = 0 \quad \text{(given)}$$
$$cos(x)\left[6cos(x) + -5\right] = 0 \quad \text{(factor)}$$
$$cos(x) = 0 \quad \text{OR} \quad 6cos(x) + -5 = 0 \quad \text{(Zero Fact Thm)}$$
$$cos(x) = 0 \quad \text{OR} \quad cos(x) = \frac{5}{6} \quad \text{(algebra)}$$

Solve

$$cos\left(x\right) = 0$$

Solution:

$$x_k = 90° + k180° \qquad \text{for } k \in \mathbb{Z}$$

Solve

$$cos\left(x\right) = \frac{5}{6}$$

Solution:

$$x_k \approx 33.557° + k360° \qquad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 326.443° + k360° \qquad \text{for } k \in \mathbb{Z}$$

19. Solve

$$cos\left(2x\right) = cos\left(6x\right)$$

Solution: note we will use the famous identity:

$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$cos\left(2x\right) = cos\left(6x\right) \qquad \text{(given)}$$

$$cos\left(2x\right) - cos\left(6x\right) = 0 \qquad \text{(Bi)}$$

$$-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right) = 0 \qquad \text{(famous id)}$$

$$-2\sin\left(4x\right)\cdot\sin\left(-2x\right) = 0$$

$$\sin\left(4x\right)\cdot\sin\left(-2x\right) = 0 \qquad \text{(divide by -2)}$$

$$\sin\left(4x\right) = 0 \quad \text{OR} \quad \sin\left(-2x\right) = 0 \qquad \text{(ZFT)}$$

Solve

$$sin\left(4x\right) = 0$$

Solution:

$$4x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx 45°k$$

Solve

$$sin\left(-2x\right) = 0$$

Solution:

$$-2x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx -90°k$$

finish solving for x and check solutions..

20. Solve

$$cos\left(-2x\right) = cos\left(6x\right)$$

Solution: note we will use the famous identity:

$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

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$$cos\left(-2x\right) = cos\left(6x\right) \quad \text{(given)}$$
$$cos\left(-2x\right) - cos\left(6x\right) = 0 \quad \text{(Bi)}$$
$$-2\sin\left(\frac{-2x+6x}{2}\right)\sin\left(\frac{-2x-6x}{2}\right) = 0 \quad \text{(famous id)}$$
$$-2\sin\left(2x\right)\cdot\sin\left(-4x\right) = 0$$
$$\sin\left(2x\right)\cdot\sin\left(-4x\right) = 0 \quad \text{(divide by -2)}$$
$$\sin\left(2x\right) = 0 \quad \text{OR} \quad \sin\left(-4x\right) = 0 \quad \text{(ZFT)}$$

Solve

$$sin\left(2x\right) = 0$$

Solution:

$$2x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx 90°k$$

Solve

$$sin\left(-4x\right) = 0$$

Solution:

$$-4x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx -45°k$$

finish solving for x and check solutions..

21. Solve

$$cos\left(-3x\right) = cos\left(5x\right)$$

Solution: note we will use the famous identity:

$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$cos\left(-3x\right) = cos\left(5x\right) \quad \text{(given)}$$
$$cos\left(-3x\right) - cos\left(5x\right) = 0 \quad \text{(Bi)}$$
$$-2\sin\left(\frac{-3x+5x}{2}\right)\sin\left(\frac{-3x-5x}{2}\right) = 0 \quad \text{(famous id)}$$
$$-2\sin\left(x\right)\cdot\sin\left(-4x\right) = 0$$
$$\sin\left(x\right)\cdot\sin\left(-4x\right) = 0 \quad \text{(divide by -2)}$$
$$\sin\left(x\right) = 0 \quad \text{OR} \quad \sin\left(-4x\right) = 0 \quad \text{(ZFT)}$$

Solve

$$sin\left(x\right) = 0$$

Solution:

$$x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx 180°k$$

Solve

$$sin\left(-4x\right) = 0$$

Solution:

$$-4x_k = 0° + k180° \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx -45°k$$

finish solving for x and check solutions..

22. Solve

$$cos\left(3x\right) = cos\left(4x\right)$$

Solution: note we will use the famous identity:

$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$cos\left(3x\right) = cos\left(4x\right) \quad \text{(given)}$$

$$cos\left(3x\right) - cos\left(4x\right) = 0 \quad \text{(Bi)}$$

$$-2\sin\left(\frac{3x+4x}{2}\right)\sin\left(\frac{3x-4x}{2}\right) = 0 \quad \text{(famous id)}$$

$$-2\sin\left(\frac{7}{2}x\right)\cdot\sin\left(-\frac{1}{2}x\right) = 0$$

$$\sin\left(\frac{7}{2}x\right)\cdot\sin\left(-\frac{1}{2}x\right) = 0 \quad \text{(divide by -2)}$$

$$\sin\left(\frac{7}{2}x\right) = 0 \quad \text{OR} \quad \sin\left(-\frac{1}{2}x\right) = 0 \quad \text{(ZFT)}$$

Solve

$$sin\left(\frac{7}{2}x\right) = 0$$

Solution:

$$\frac{7}{2}x_k = 0^\circ + k180^\circ \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx 51.43^\circ k$$

Solve

$$sin\left(-\frac{1}{2}x\right) = 0$$

Solution:

$$-\frac{1}{2}x_k = 0^\circ + k180^\circ \qquad \text{for } k \in \mathbb{Z}$$

$$x_k \approx -360^\circ k$$

finish solving for x and check solutions..

23. Find all solutions

$$\cos x = 2\cos^2 x$$

Solution: do NOT divide.. dangerous.. instead..

$$\cos x = 2\cos^2 x \quad \text{(given)}$$

$$\cos x - 2\cos^2 x = 0 \quad \text{(algebra)}$$

$$\cos x(1 - 2\cos x) = 0 \quad \text{(algebra, factor)}$$

$$\cos x = 0 \qquad 1 - 2\cos x = 0 \quad \text{(Zero Factor Theorem)}$$

$$\cos x = 0 \qquad \cos x = 1/2 \quad \text{(algebra..)}$$

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then...
The set of all real solutions to $\cos x = 0$ is of the form...

$$x = 90° + k180°$$

said differently...

$$x = \ldots, -90°, 90°, 270°, 450°, \ldots$$

AND
The set of all real solutions to $\cos x = .5$ is of the form...

$$x = 60.0° + k360° \qquad \text{or} \quad x = 300.0° + k360°$$

said differently...

$$x = \ldots, -60.0°, 60.0°, 300.0°, 420.0°, \ldots$$

24. Find all solutions

$$\cos x = 1 - \sin^2 x$$

Solution: ONE way to look at it..

$$\begin{aligned}
\cos x &= 1 - \sin^2 x && \text{(given)} \\
&&& \text{(famous idea change all to cosines...)} \\
\cos x &= \cos^2 x && \text{(pythagoras ID)} \\
\cos x - \cos^2 x &= 0 && \text{(algebra)} \\
\cos x(1 - \cos x) &= 0 && \text{(algebra, factor)} \\
\cos x = 0 \qquad 1 - \cos x &= 0 && \text{(Zero Factor Theorem)} \\
\cos x = 0 \qquad \cos x &= 1 && \text{(algebra..)}
\end{aligned}$$

then...
The set of all real solutions to $\cos x = 0$ is of the form...

$$x = 90° + k180°$$

said differently...

$$x = \ldots, -90°, 90°, 270°, 450°, \ldots$$

AND The set of all real solutions to $\cos x = 1$ is of the form...

$$x = 0.0° + k360°$$

said differently...

$$x = \ldots, -360.0°, 0.0°, 360.0°, 720.0°, \ldots$$

25. Find all solutions

$$\tan x = \sin x$$

Solution: ONE way to look at it..

$$\tan x = \sin x \quad \text{(given)}$$

(famous idea move all to one side, try to factor.., change to sines n cosines.)

$$\tan x - \sin x = 0 \quad \text{(algebra)}$$

$$\frac{\sin x}{\cos x} - \sin x = 0 \quad \text{(IDS)}$$

$$\sin x\left(\frac{1}{\cos x} - 1\right) = 0 \quad \text{(algebra, factor)}$$

$$\sin x = 0 \qquad \frac{1}{\cos x} - 1 = 0 \quad \text{(Zero Factor Theorem)}$$

$$\sin x = 0 \qquad \cos x = 1 \quad \text{(algebra.., note had to mult by } \cos x\text{)}$$

then...
The set of all real solutions to $\sin x = 0$ is of the form...

$$x = 180°k$$

said differently...

$$x = \ldots, -180°, 0°, 180°, 360°, \ldots$$

AND The set of all real solutions to $\cos x = 1$ is of the form...

$$x = 0.0° + k360°$$

said differently...

$$x = \ldots, -360.0°, 0.0°, 360.0°, 720.0°, \ldots$$

HOWEVER, because we mult both by $\cos x$, extraneous solutions may have been introduced so each of these solutions should be checked and extraneous solutions need to be discarded.

26. Find all solutions

$$\cos(3x + \pi) = \frac{-1}{2}$$

27. Find all solutions

$$\cos(40° - 2x) = \frac{-1}{3}$$

28. Find all solutions

$$\frac{4}{\sec x - 1} - 1 = \sec x$$

29. Find all solutions

$$\csc(2x) = -\sin^2 + 1$$

30. Find all solutions

$$\sin(2x) = \cos(2x)$$

31. Find all solutions

$$\sin(4x) = \cos(2x)$$

32. Find all solutions

$$\cos(5x) = \cos(7x)$$

Solution: ONE way to look at it.. is to move all to one side, set to zero.. then try to to change the difference to a product.. use famous identities.. then use zero factor theorem..

33. (xtra fun..take your time on this one) Find all solutions

$$\sin(5x) = \cos(7x)$$

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