

Solving Trig Equations: The Others

Main Idea

In the last sections, we learned to solve very basic trig equations as well as slight variations of these basic ones. In this section, we show techniques which can be used to turn other equations into basic ones. The use of identities will be essential. The general strategy is to get all expressions on one side of the equation and 0 on the other side. Then we try to factor the expressions into simpler terms so that we may use the Zero Factor Theorem. This is, generally speaking, easier said than done. Yet for our class, it may be appropriate to propose mostly equations which do in fact factor in a relatively easy way, so long as we have full command and use of the famous identities.

The following suggestions may often prove helpful.

Solving Trig Eqs: Some Strategies

- Tweak to turn into ez ones.
- Turn everything into sines or cosines.
- Reduce angles so they are all equal.
- Use the famous identities
- Use conjugates
- See if you can factor as a quadratic equation
- Careful not to mult/or divide by zero
- Check answers when finished.

Example:

Solve

$$24\sin^2(x) + -38\sin(x) + 15 = 0$$

$$\begin{aligned}
 24\sin^2(x) + -38\sin(x) + 15 &= 0 && \text{(given)} \\
 [4\sin(x) + -3] \cdot [6\sin(x) + -5] &= 0 && \text{(factor)} \\
 4\sin(x) + -3 = 0 \quad \text{OR} \quad 6\sin(x) + -5 &= 0 && \text{(Zero Fact Thm)} \\
 \sin(x) = \frac{3}{4} \quad \text{OR} \quad \sin(x) = \frac{5}{6} &&& \text{(algebra)}
 \end{aligned}$$

Solve

$$\sin(x) = \frac{3}{4}$$

Solution:

$$x_k \approx 48.59^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 131.41^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

Solve

$$\sin(x) = \frac{5}{6}$$

Solution:

$$x_k \approx 56.443^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 123.557^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

Example:

Solve

$$6\cos^2(x) + -13\cos(x) + 5 = 0$$

$$\begin{aligned}
 6\cos^2(x) + -13\cos(x) + 5 &= 0 && \text{(given)} \\
 [2\cos(x) + -1] \cdot [3\cos(x) + -5] &= 0 && \text{(factor)} \\
 2\cos(x) + -1 = 0 \quad \text{OR} \quad 3\cos(x) + -5 &= 0 && \text{(Zero Fact Thm)} \\
 \cos(x) = \frac{1}{2} \quad \text{OR} \quad \cos(x) = \frac{5}{3} &&& \text{(algebra)}
 \end{aligned}$$

Solve

$$\cos(x) = \frac{1}{2}$$

Solve

Solution:

$$\cos(x) = \frac{5}{3}$$

$$x_k \approx 60^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 300^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

no real solution for x

Example:

Solve

$$2\sin^2(x) + -5\sin(x) + -3 = 0$$

$$\begin{aligned}
 2\sin^2(x) + -5\sin(x) + -3 &= 0 && \text{(given)} \\
 [2\sin(x) + 1] \cdot [1\sin(x) + -3] &= 0 && \text{(factor)} \\
 2\sin(x) + 1 = 0 \quad \text{OR} \quad 1\sin(x) + -3 &= 0 && \text{(Zero Fact Thm)} \\
 \sin(x) = -\frac{1}{2} \quad \text{OR} \quad \sin(x) = 3 &&& \text{(algebra)}
 \end{aligned}$$

Solve

$$\sin(x) = -\frac{1}{2}$$

Solve

Solution:

$$\sin(x) = 3$$

$$x_k \approx -30^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

OR

$$x_k \approx 210^\circ + k360^\circ \quad \text{for } k \in \mathbb{Z}$$

no real solution for x

Solving Trig Equations: The Others

1. Solve

$$3\sin^2(x) + \sin(x) = 0$$

2. Solve

$$2\sin^2(x) + 5\sin(x) = 0$$

3. Solve

$$5\cos^2(x) + \cos(x) = 0$$

4. Solve

$$6\cos^2(x) + 2\cos(x) = 0$$

5. Solve

$$8\sin^2(x) + -2\sin(x) + -3 = 0$$

6. Solve

$$4\sin^2(x) + 0\sin(x) + -1 = 0$$

7. Solve

$$4\cos^2(x) + 0\cos(x) + -1 = 0$$

8. Solve

$$6\cos^2(x) + -1\cos(x) + -1 = 0$$

9. Solve

$$12\sin^2(x) + -5\sin(x) + -3 = 0$$

10. Solve

$$4\sin^2(x) + 8\sin(x) + -5 = 0$$

11. Solve

$$10\cos^2(x) + -18\cos(x) + -4 = 0$$

12. Solve

$$60\cos^2(x) + 2\cos(x) + -6 = 0$$

13. Solve

$$3 + \sin(x) = 3\cos^2(x)$$

14. Solve

$$3 + -\cos(x) = 3\sin^2(x)$$

15. Solve

$$2 + \sin(x) = 2\cos^2(x)$$

16. Solve

$$2 + -3\sin(x) = 2\cos^2(x)$$

17. Solve

$$4 + -3\cos(x) = 4\sin^2(x)$$

18. Solve

$$6 + -5\cos(x) = 6\sin^2(x)$$

19. Solve

$$\cos(2x) = \cos(6x)$$

20. Solve

$$\cos(-2x) = \cos(6x)$$

21. Solve

$$\cos(-3x) = \cos(5x)$$

22. Solve

$$\cos(3x) = \cos(4x)$$

23. Find all solutions

$$\cos x = 2\cos^2 x$$

24. Find all solutions

$$\cos x = 1 - \sin^2 x$$

25. Find all solutions

$$\tan x = \sin x$$

26. Find all solutions

$$\cos(3x + \pi) = \frac{-1}{2}$$

27. Find all solutions

$$\cos(40^\circ - 2x) = \frac{-1}{3}$$

28. Find all solutions

$$\frac{4}{\sec x - 1} - 1 = \sec x$$

29. Find all solutions

$$\csc(2x) = -\sin^2 + 1$$

30. Find all solutions

$$\sin(2x) = \cos(2x)$$

31. Find all solutions

$$\sin(4x) = \cos(2x)$$

32. Find all solutions

$$\cos(5x) = \cos(7x)$$

33. (xtra fun..take your time on this one) Find all solutions

$$\sin(5x) = \cos(7x)$$