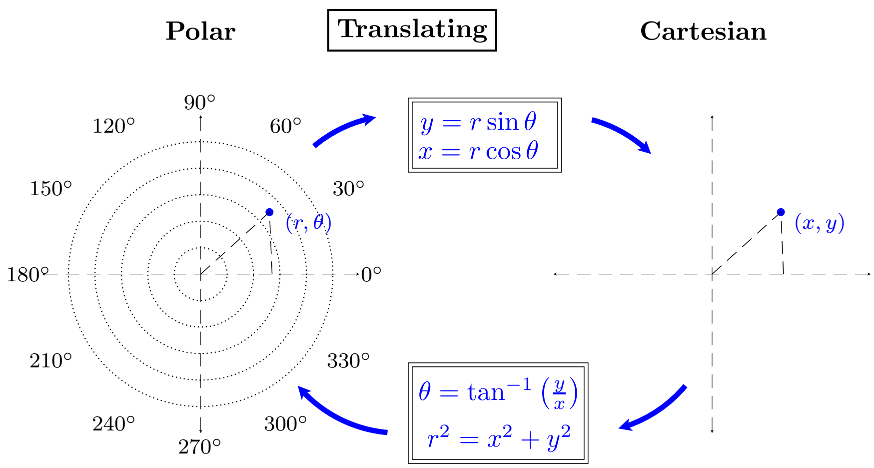
Converting Equations Polar & Cartesian

The IDEA

In the last couple sections we've established, among other things, a 'dictionary' to convert coordinate points from cartesian to polar and vise-varsa. In this section, we would like to exploit that same dictionary to convert equations. As a reminder, the dictionary has two components, 'from' polar, and 'to' polar. The goal in converting, in most cases, is to eliminate all x's and y's and replace them with appropriate expressions involving r's and θ's if we are translating to polar. When converting from polar, the idea is to eliminate the r's and θ's. We summarize the 'dictionary' below.



Example: Converting Equation of a line to Polar

Convert the following equation to polar coordinates:

$$y = x$$

Solution: Before we get started, let us recognize that there is not a 1-1 correspondence between polar and cartesian coordinates, that is for each cartesian coordinates, there may be infinite polar coordinates corresponding to the same point. In light of this, the 'translating' of this equation is not unique. Observe,

$$y = x \qquad \text{(given)}$$

$$\frac{y}{x} = 1 \qquad \text{(div by } x\text{, setting up to use translating dictionary, } \tan\theta = y/x\text{)}$$

$$\tan\theta = 1 \qquad \text{(used } \tan\theta = y/x\text{)}$$

$$\theta = 45° \qquad \text{(notice: this there are infinite possibilities for } \theta \text{ (ie } -135°, 225°\text{...), this is just one of them.)}$$

Thus, one way to translate the equation $y = x$ to polar is to convert to $\theta = 45°$

Example: Converting Equation of a circle to Polar

Convert the following equation to polar coordinates:

$$x^2 + y^2 = 25$$

Solution: Before we get started, let us recognize that there is not a 1-1 correspondence between polar and cartesian coordinates, that is for each cartesian coordinates, there may be infinite polar coordinates corresponding to the same point. In light of this, the 'translating' of this equation is not unique. Observe,

$$\begin{aligned} x^2+y^2 &= 25 && \text{(given)} \\ r^2 &= 25 && \text{(used the dictionary } r^2=x^2+y^2\text{)} \end{aligned}$$

at this point we have eliminated all x's and y's from the equation, thus the converting is complete. It is sometimes desirable to solve for r explicitly, thus we could also say $r=5$ or $r=-5$, in fact all three of these equations, $r^2=25$, $r=5$, AND $r=-5$ all yield the same graph as $x^2+y^2=25$.

Example: Converting the equation of a circle to Polar

Convert the following equation to polar coordinates:

$$x^2+3x+5+y^2-4y+2=25$$

Solution:

$$\begin{aligned} x^2+3x+5+y^2-4y+2 &= 25 && \text{(given)} \\ (x^2+y^2)+3(x)-4(y) &= 18 && \text{(just cleaning up, getting ready to use the dictionary)} \\ (r^2)+3(r\cos\theta)-4(r\sin\theta) &= 18 && \text{(used the dictionary)} \\ r^2+3r\cos\theta-4r\sin\theta &= 18 && \text{(algebra)} \end{aligned}$$

Example: Converting the equation of a circle from Polar

Convert the following equation of a circle to polar coordinates:

$$r=12\cos\theta$$

Solution:

$$\begin{aligned} r &= 12\cos\theta && \text{(given)} \\ r\cdot r &= r(12\cos\theta) && \text{(clever little idea, mult both sides by } r\text{)} \\ r^2 &= 12(r\cos\theta) && \text{(algebra, getting ready to use dictionary)} \\ x^2+y^2 &= 12x && \text{(used dictionary, all } r\text{'s and } \theta\text{'s gone!!)} \\ y^2+x^2+-12x &= 0 && \text{(optional step: complete the square)} \\ y^2+x^2+-12x+36 &= 36 && \text{(optional to step: complete the square)} \\ y^2+\left(x+-6\right)^2 &= 36 && \text{(Shows its a circle, shows center/radius)} \end{aligned}$$

Converting Equations Polar & Cartesian

1. Convert the following equation to polar coordinates:

$$y = 2x$$

Solution: Note: the 'translating' of this equation is not unique.

$$\begin{aligned} y &= 2x && \text{(given)} \\ \frac{y}{x} &= 2 && \text{(div by } x\text{, setting up to use translating dictionary, } \tan\theta = y/x) \\ \tan\theta &= 2 && \text{(used } \tan\theta = y/x) \\ \theta &\approx 63.435^\circ && \text{(notice: above eq has infinite solutions for } \theta\text{, this is just one of them.)} \end{aligned}$$

Thus, one way to translate the equation $y = 2x$ to polar is to convert to $\theta \approx 63.435^\circ$

2. Convert the following equation to polar coordinates:

$$y = -\frac{2}{3}x$$

Solution: Note: the 'translating' of this equation is not unique.

$$\begin{aligned} y &= -\frac{2}{3}x && \text{(given)} \\ \frac{y}{x} &= -\frac{2}{3} && \text{(div by } x\text{, setting up to use translating dictionary, } \tan\theta = y/x) \\ \tan\theta &= -\frac{2}{3} && \text{(used } \tan\theta = y/x) \\ \theta &\approx -33.69^\circ && \text{(notice: above eq has infinite solutions for } \theta\text{, this is just one of them.)} \end{aligned}$$

Thus, one way to translate the equation $y = 2x$ to polar is to convert to $\theta \approx -33.69^\circ$

3. Convert the following equation to polar coordinates:

$$y = \frac{2}{5}x$$

Solution: Note: the 'translating' of this equation is not unique.

$$y = \frac{2}{5}x \qquad \text{(given)}$$
$$\frac{y}{x} = \frac{2}{5} \qquad \text{(div by } x\text{, setting up to use translating dictionary, } \tan\theta = y/x)$$
$$\tan\theta = \frac{2}{5} \qquad \text{(used } \tan\theta = y/x)$$
$$\theta \approx 21.801° \qquad \text{(notice: above eq has infinite solutions for } \theta\text{, this is just one of them.)}$$

Thus, one way to translate the equation $y = 2x$ to polar is to convert to $\theta \approx 21.801°$

4. Convert the following equation to polar coordinates:

$$y = \frac{4}{3}x$$

Solution: Note: the 'translating' of this equation is not unique.

$$y = \frac{4}{3}x \qquad \text{(given)}$$
$$\frac{y}{x} = \frac{4}{3} \qquad \text{(div by } x\text{, setting up to use translating dictionary, } \tan\theta = y/x)$$
$$\tan\theta = \frac{4}{3} \qquad \text{(used } \tan\theta = y/x)$$
$$\theta \approx 53.13° \qquad \text{(notice: above eq has infinite solutions for } \theta\text{, this is just one of them.)}$$

Thus, one way to translate the equation $y = 4x$ to polar is to convert to $\theta \approx 53.13°$

5. Convert the following equation to polar coordinates:

$$y = -\frac{4}{3}x$$

Solution: Note: the 'translating' of this equation is not unique.

$$y = -\frac{4}{3}x \qquad \text{(given)}$$
$$\frac{y}{x} = -\frac{4}{3} \qquad \text{(div by } x\text{, setting up to use translating dictionary, } \tan\theta = y/x)$$
$$\tan\theta = -\frac{4}{3} \qquad \text{(used } \tan\theta = y/x)$$
$$\theta \approx -53.13° \qquad \text{(notice: above eq has infinite solutions for } \theta\text{, this is just one of them.)}$$

Thus, one way to translate the equation $y = -4x$ to polar is to convert to $\theta \approx -53.13°$

6. Convert the following equation of a circle to polar coordinates:

$$2x^2 + 3x + 2y^2 + -5y = 7$$

Solution:

$$\begin{aligned} 2x^2 + 3x + 2y^2 + -5y &= 7 && \text{(given)} \\ 2\left(x^2 + y^2\right) + 3(x) + -5(y) &= 7 && \text{(ready to use the dictionary....)} \\ 2\left(r^2\right) + 3(r\cos\theta) + -5(r\sin\theta) &= 7 && \text{(used the dictionary)} \end{aligned}$$

7. Convert the following equation of a circle to polar coordinates:

$$-5x^2 + 2x + -5y^2 + 7y = 25$$

Solution:

$$\begin{aligned} -5x^2 + 2x + -5y^2 + 7y &= 25 && \text{(given)} \\ -5\left(x^2 + y^2\right) + 2(x) + 7(y) &= 25 && \text{(ready to use the dictionary....)} \\ -5\left(r^2\right) + 2(r\cos\theta) + 7(r\sin\theta) &= 25 && \text{(used the dictionary)} \end{aligned}$$

8. Convert the following equation of a circle to polar coordinates:

$$4x^2 + \frac{3}{2}x + 4y^2 + 1y = 9$$

Solution:

$$\begin{aligned} 4x^2 + \frac{3}{2}x + 4y^2 + 1y &= 9 && \text{(given)} \\ 4\left(x^2 + y^2\right) + \frac{3}{2}(x) + 1(y) &= 9 && \text{(ready to use the dictionary....)} \\ 4\left(r^2\right) + \frac{3}{2}(r\cos\theta) + 1(r\sin\theta) &= 9 && \text{(used the dictionary)} \end{aligned}$$

9. Convert the following equation of a circle to polar coordinates:

$$3x^2 + 3x + 3y^2 + 5y = 4$$

Solution:

$$\begin{aligned} 3x^2 + 3x + 3y^2 + 5y &= 4 && \text{(given)} \\ 3\left(x^2 + y^2\right) + 3(x) + 5(y) &= 4 && \text{(ready to use the dictionary....)} \\ 3\left(r^2\right) + 3(r\cos\theta) + 5(r\sin\theta) &= 4 && \text{(used the dictionary)} \end{aligned}$$

10. Convert the following equation of a circle to polar coordinates:

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

Solution:

$$\frac{x^2}{9} + \frac{y^2}{25} = 1 \qquad \text{(given)}$$

$$\frac{(r\cos\theta)^2}{9} + \frac{(r\sin\theta)^2}{25} = 1 \qquad \text{(ready to use the dictionary....)}$$

$$r^2\left(\frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{25}\right) = 1 \qquad \text{(alebra)}$$

11. Convert the following equation of a circle to polar coordinates:

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

Solution:

$$\frac{x^2}{4} + \frac{y^2}{16} = 1 \qquad \text{(given)}$$

$$\frac{(r\cos\theta)^2}{4} + \frac{(r\sin\theta)^2}{16} = 1 \qquad \text{(ready to use the dictionary....)}$$

$$r^2\left(\frac{\cos^2\theta}{4} + \frac{\sin^2\theta}{16}\right) = 1 \qquad \text{(alebra)}$$

12. Convert the following equation of a circle to polar coordinates:

$$\frac{x^2}{4} + \frac{y^2}{49} = 1$$

Solution:

$$\frac{x^2}{4} + \frac{y^2}{49} = 1 \qquad \text{(given)}$$

$$\frac{(r\cos\theta)^2}{4} + \frac{(r\sin\theta)^2}{49} = 1 \qquad \text{(ready to use the dictionary....)}$$

$$r^2\left(\frac{\cos^2\theta}{4} + \frac{\sin^2\theta}{49}\right) = 1 \qquad \text{(alebra)}$$

13. Convert the following equation of a circle to polar coordinates:

$$r = 2\sin\theta$$

Solution:

$$\begin{aligned}
r &= 2\sin\theta && \text{(given)}\\
r\cdot r &= r(2\sin\theta) && \text{(clever little idea, mult both sides by } r\text{)}\\
r^2 &= 2(r\sin\theta) && \text{(algebra, getting ready to use dictionary)}\\
x^2+y^2 &= 2y && \text{(used dictionary, all } r\text{'s and } \theta\text{'s gone!!)}\\
x^2+y^2+-2y &= 0 && \text{(optional step: complete the square)}\\
x^2+y^2+-2y+1 &= 1 && \text{(optional to step: complete the square)}\\
x^2+\left(y+-1\right)^2 &= 1 && \text{(Shows its a circle, shows center/radius)}
\end{aligned}$$

14. Convert the following equation of a circle to polar coordinates:

$$r = 5\sin\theta$$

Solution:

$$\begin{aligned}
r &= 5\sin\theta && \text{(given)}\\
r\cdot r &= r(5\sin\theta) && \text{(clever little idea, mult both sides by } r\text{)}\\
r^2 &= 5(r\sin\theta) && \text{(algebra, getting ready to use dictionary)}\\
x^2+y^2 &= 5y && \text{(used dictionary, all } r\text{'s and } \theta\text{'s gone!!)}\\
x^2+y^2+-5y &= 0 && \text{(optional step: complete the square)}\\
x^2+y^2+-5y+\frac{25}{4} &= \frac{25}{4} && \text{(optional to step: complete the square)}\\
x^2+\left(y+-\frac{5}{2}\right)^2 &= \frac{25}{4} && \text{(Shows its a circle, shows center/radius)}
\end{aligned}$$

15. Convert the following equation of a circle to polar coordinates:

$$r = 6\sin\theta$$

Solution:

$$r = 6\sin\theta \quad \text{(given)}$$
$$r \cdot r = r(6\sin\theta) \quad \text{(clever little idea, mult both sides by } r\text{)}$$
$$r^2 = 6(r\sin\theta) \quad \text{(algebra, getting ready to use dictionary)}$$
$$x^2 + y^2 = 6y \quad \text{(used dictionary, all } r\text{'s and } \theta\text{'s gone!!)}$$
$$x^2 + y^2 + -6y = 0 \quad \text{(optional step: complete the square)}$$
$$x^2 + y^2 + -6y + 9 = 9 \quad \text{(optional to step: complete the square)}$$
$$x^2 + \left(y + -3\right)^2 = 9 \quad \text{(Shows its a circle, shows center/radius)}$$

16. Convert the following equation of a circle to polar coordinates:

$$r = 11\sin\theta$$

Solution:

$$r = 11\sin\theta \quad \text{(given)}$$
$$r \cdot r = r(11\sin\theta) \quad \text{(clever little idea, mult both sides by } r\text{)}$$
$$r^2 = 11(r\sin\theta) \quad \text{(algebra, getting ready to use dictionary)}$$
$$x^2 + y^2 = 11y \quad \text{(used dictionary, all } r\text{'s and } \theta\text{'s gone!!)}$$
$$x^2 + y^2 + -11y = 0 \quad \text{(optional step: complete the square)}$$
$$x^2 + y^2 + -11y + \frac{121}{4} = \frac{121}{4} \quad \text{(optional to step: complete the square)}$$
$$x^2 + \left(y + -\frac{11}{2}\right)^2 = \frac{121}{4} \quad \text{(Shows its a circle, shows center/radius)}$$

17. Convert the following equation of a circle to polar coordinates:

$$r = 8\cos\theta$$

Solution:

$$r = 8\cos\theta \quad \text{(given)}$$
$$r \cdot r = r(8\cos\theta) \quad \text{(clever little idea, mult both sides by } r\text{)}$$
$$r^2 = 8(r\cos\theta) \quad \text{(algebra, getting ready to use dictionary)}$$
$$x^2 + y^2 = 8x \quad \text{(used dictionary, all } r\text{'s and } \theta\text{'s gone!!)}$$
$$y^2 + x^2 + -8x = 0 \quad \text{(optional step: complete the square)}$$
$$y^2 + x^2 + -8x + 16 = 16 \quad \text{(optional to step: complete the square)}$$
$$y^2 + \left(x + -4\right)^2 = 16 \quad \text{(Shows its a circle, shows center/radius)}$$

18. Convert the following equation of a circle to polar coordinates:

$$r = 12\cos\theta$$

Solution:

$r = 12\cos\theta$	(given)
$r \cdot r = r(12\cos\theta)$	(clever little idea, mult both sides by r)
$r^2 = 12(r\cos\theta)$	(algebra, getting ready to use dictionary)
$x^2 + y^2 = 12x$	(used dictionary, all r's and θ's gone!!)
$y^2 + x^2 + -12x = 0$	(optional step: complete the square)
$y^2 + x^2 + -12x + 36 = 36$	(optional to step: complete the square)
$y^2 + \left(x + -6\right)^2 = 36$	(Shows its a circle, shows center/radius)

19. Convert the following equation of a circle to polar coordinates:

$$r = 5\cos\theta$$

Solution:

$r = 5\cos\theta$	(given)
$r \cdot r = r(5\cos\theta)$	(clever little idea, mult both sides by r)
$r^2 = 5(r\cos\theta)$	(algebra, getting ready to use dictionary)
$x^2 + y^2 = 5x$	(used dictionary, all r's and θ's gone!!)
$y^2 + x^2 + -5x = 0$	(optional step: complete the square)
$y^2 + x^2 + -5x + \frac{25}{4} = \frac{25}{4}$	(optional to step: complete the square)
$y^2 + \left(x + -\frac{5}{2}\right)^2 = \frac{25}{4}$	(Shows its a circle, shows center/radius)

20. Convert the following equation of a circle to polar coordinates:

$$r = 11\cos\theta$$

Solution:

$r = 11\cos\theta$	(given)
$r \cdot r = r(11\cos\theta)$	(clever little idea, mult both sides by r)
$r^2 = 11(r\cos\theta)$	(algebra, getting ready to use dictionary)
$x^2 + y^2 = 11x$	(used dictionary, all r's and θ's gone!!)
$y^2 + x^2 + -11x = 0$	(optional step: complete the square)
$y^2 + x^2 + -11x + \frac{121}{4} = \frac{121}{4}$	(optional to step: complete the square)
$y^2 + \left(x + -\frac{11}{2}\right)^2 = \frac{121}{4}$	(Shows its a circle, shows center/radius)