More onComplex Numbers

The Idea

Last section was 'out of this world'. We introduced the imaginary number, their standard form '$a + bi$', we introduced their home, the complex plane, and we introduced some simple arithmetic operations on them such as adding/multiplying. In this section, we continue on the same theme, adding to that some division skills, we add some some famous terminology, such as 'conjugates', and we look further into the calculation of many exponential powers of i.

How to divide in the $\mathbb{C}$-world The layman way to divide.

The key lies in the observation that multiplying pairs of conjugate complex numbers always yields real numbers. In a way, it is sort of a way to smack a complex number on its head and turn it into a real number, sort of. Every complex number has a conjugate defined as follows, when written in standard form, the conjugate of $a+bi$ is $a-bi$. In other words, the conjugate of a complex number is the same number with the sign of the complex part switched. Now, observe how the product of conjugates *always* yields a real number. Take, for example, the complex number $2+3i$, its conjugate is $2-3i$:

$$\begin{aligned}(2+3i)(2-3i) &= 4+6i-6i-9\cdot i^2 && \text{(FOIL)}\\ &= 4+0-9\cdot(-1) && \text{(BI)}\\ &= 13 && \text{(as promised, a real number)}\end{aligned}$$

Now, we see how this will help us divide. Suppose we want to divide $\frac{3+5i}{2+3i}$

Divide

$$\frac{5i+3}{3i+2}$$

$$\begin{aligned}\frac{5i+3}{3i+2} &= \frac{5i+3}{3i+2}\cdot 1 && \text{(MiD)}\\ &= \frac{5i+3}{3i+2}\cdot\frac{-3i+2}{-3i+2} && \text{(JOT)}\\ &= \frac{-15i^2+i+6}{-9i^2+4} && \text{(MAT, FOIL)}\\ &= \frac{i+21}{13} && \text{(BI)}\\ &= \frac{21}{13}+\frac{i}{13} && \text{(BI)}\end{aligned}$$

Here is another example,

Divide

$$\frac{5i-3}{i+3}$$

$$
\begin{aligned}
\frac{5i-3}{i+3} &= \frac{5i-3}{i+3}\cdot 1 && \text{(MiD)}\\
&= \frac{5i-3}{i+3}\cdot\frac{-i+3}{-i+3} && \text{(JOT)}\\
&= \frac{-5i^2+18i-9}{-i^2+9} && \text{(MAT, FOIL)}\\
&= \frac{18i-4}{10} && \text{(BI)}\\
&= \frac{-4}{10}+\frac{18i}{10} && \text{(BI)}
\end{aligned}
$$

How to divide in the $\mathbb{C}$-world

As usual, to divide means to multiply by the multiplicative inverse. Thus, we need and want to address this question: for any non-zero complex number, $a+bi$ what is its multiplicative inverse? We claim the inverse is $\frac{a-bi}{a^2+b^2}$. To check this we simply check that their product is 1.

Multiplicative Inverses in **C**

$$
\begin{aligned}
(a+bi)\left(\frac{a-bi}{a^2+b^2}\right) &= \frac{(a+bi)}{1}\left(\frac{a-bi}{a^2+b^2}\right)\\
&= \frac{a^2+abi-abi-bi^2}{a^2+b^2}\\
&= = \frac{a^2+b^2}{a^2+b^2} = 1
\end{aligned}
$$

Example Dividing in **C** :

$$
\begin{aligned}
(3-2i)\div(1+3i) &= (3-2i)\cdot\frac{1-3i}{1^2+3^2}\\
&= \frac{3-9i-2i+6i^2}{10}\\
&= \frac{-3-11i}{10} = \frac{-3}{10}-\frac{11}{10}i
\end{aligned}
$$

Another way to 'divide' and in essence carry out the same computation is to multiply numerator and denominator by the conjugate of the denominator. For example, if the denominator is $a+bi$, then multiplying both numerator and denominator will annihilate the i's on the denominator. This is a very popular method of 'dividing. For example.

Compute w/ Complex Numbers Calculate and write in standard form.

$$\frac{2i+1}{i+1}$$

There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\begin{aligned}\frac{2i+1}{i+1} &= 2i+1 \cdot \frac{-i+1}{1^2+1^2} \\ &= \frac{2i+1-i+1}{2} \\ &= \frac{i+3}{2} \\ &= \frac{3}{2}+\frac{1}{2}i\end{aligned}$$

Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{2i+1}{i+1} &= \frac{2i+1}{i+1}\cdot 1 \\ &= \frac{2i+1}{i+1}\cdot\frac{-i+1}{-i+1} \\ &= \frac{i+3}{2} \\ &= \frac{3}{2}+\frac{1}{2}i\end{aligned}$$

Compute w/ Complex Numbers Calculate and write in standard form.

$$\frac{2i+3}{5i-2}$$

There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\begin{aligned}\frac{2i+3}{5i-2} &= 2i+3\cdot\frac{-5i-2}{-2^2+5^2} \\ &= \frac{2i+3-5i-2}{29} \\ &= \frac{-19i+4}{29} \\ &= \frac{4}{29}+\frac{-19}{29}i\end{aligned}$$

Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{2i+3}{5i-2} &= \frac{2i+3}{5i-2}\cdot 1 \\ &= \frac{2i+3}{5i-2}\cdot\frac{-5i-2}{-5i-2} \\ &= \frac{-19i+4}{29} \\ &= \frac{4}{29}+\frac{-19}{29}i\end{aligned}$$

Instructions

1. Divide

$$\frac{7i+1}{7i+3}$$

Solution:

$$\begin{aligned}
\frac{7i+1}{7i+3} &= \frac{7i+1}{7i+3}\cdot 1 && \text{(MiD)}\\
&= \frac{7i+1}{7i+3}\cdot\frac{-7i+3}{-7i+3} && \text{(JOT)}\\
&= \frac{-49i^2+14i+3}{-49i^2+9} && \text{(MAT, FOIL)}\\
&= \frac{14i+52}{58} && \text{(BI)}\\
&= \frac{52}{58}+\frac{14i}{58} && \text{(BI)}
\end{aligned}$$

2. Divide

$$\frac{4i+2}{-5i+2}$$

Solution:

$$\begin{aligned}
\frac{4i+2}{-5i+2} &= \frac{4i+2}{-5i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{4i+2}{-5i+2}\cdot\frac{5i+2}{5i+2} && \text{(JOT)}\\
&= \frac{20i^2+18i+4}{-25i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{18i-16}{29} && \text{(BI)}\\
&= \frac{-16}{29}+\frac{18i}{29} && \text{(BI)}
\end{aligned}$$

3. Divide

$$\frac{5i+2}{4i+2}$$

Solution:

$$\begin{aligned}
\frac{5i+2}{4i+2} &= \frac{5i+2}{4i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{5i+2}{4i+2}\cdot\frac{-4i+2}{-4i+2} && \text{(JOT)}\\
&= \frac{-20i^2+2i+4}{-16i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{2i+24}{20} && \text{(BI)}\\
&= \frac{24}{20}+\frac{2i}{20} && \text{(BI)}
\end{aligned}$$

4. Divide

$$\frac{3i+1}{2i+1}$$

Solution:

$$\begin{aligned}
\frac{3i+1}{2i+1} &= \frac{3i+1}{2i+1}\cdot 1 && \text{(MiD)}\\
&= \frac{3i+1}{2i+1}\cdot\frac{-2i+1}{-2i+1} && \text{(JOT)}\\
&= \frac{-6i^2+i+1}{-4i^2+1} && \text{(MAT, FOIL)}\\
&= \frac{i+7}{5} && \text{(BI)}\\
&= \frac{7}{5}+\frac{i}{5} && \text{(BI)}
\end{aligned}$$

5. Divide

$$\frac{5i+2}{i+2}$$

Solution:

$$\begin{aligned}
\frac{5i+2}{i+2} &= \frac{5i+2}{i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{5i+2}{i+2}\cdot\frac{-i+2}{-i+2} && \text{(JOT)}\\
&= \frac{-5i^2+8i+4}{-i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{8i+9}{5} && \text{(BI)}\\
&= \frac{9}{5}+\frac{8i}{5} && \text{(BI)}
\end{aligned}$$

6. Divide

$$\frac{7i+2}{3i+2}$$

Solution:

$$\begin{aligned}
\frac{7i+2}{3i+2} &= \frac{7i+2}{3i+2}\cdot 1 && \text{(MiD)}\\
&= \frac{7i+2}{3i+2}\cdot\frac{-3i+2}{-3i+2} && \text{(JOT)}\\
&= \frac{-21i^2+8i+4}{-9i^2+4} && \text{(MAT, FOIL)}\\
&= \frac{8i+25}{13} && \text{(BI)}\\
&= \frac{25}{13}+\frac{8i}{13} && \text{(BI)}
\end{aligned}$$

7. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{2i+1}{i+1}$$

Solution: There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\begin{aligned}
\frac{2i+1}{i+1} &= 2i+1 \cdot \frac{-i+1}{1^2+1^2} \\
&= \frac{2i+1-i+1}{2} \\
&= \frac{i+3}{2} \\
&= \frac{3}{2} + \frac{1}{2}i
\end{aligned}$$

Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}
\frac{2i+1}{i+1} &= \frac{2i+1}{i+1} \cdot 1 \\
&= \frac{2i+1}{i+1} \cdot \frac{-i+1}{-i+1} \\
&= \frac{i+3}{2} \\
&= \frac{3}{2} + \frac{1}{2}i
\end{aligned}$$

8. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{2i+1}{2i+3}$$

Solution: There are at least a couple ways to go about this.. one way, to note 'divide' means 'multiply by inverse'... so..

$$\begin{aligned}
\frac{2i+1}{2i+3} &= 2i+1 \cdot \frac{-2i+3}{3^2+2^2} \\
&= \frac{2i+1-2i+3}{13} \\
&= \frac{4i+7}{13} \\
&= \frac{7}{13} + \frac{4}{13}i
\end{aligned}$$

Another way to do it.. (more popular) is to simply multiply numerator and denominator by the conjugate of the denominator.

$$\begin{aligned}\frac{2i+1}{2i+3} &= \frac{2i+1}{2i+3}\cdot 1\\ &= \frac{2i+1}{2i+3}\cdot\frac{-2i+3}{-2i+3}\\ &= \frac{4i+7}{13}\\ &= \frac{7}{13}+\frac{4}{13}i\end{aligned}$$

9. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{1}{i}$$

Solution: hint: the bottom is $0+1i$

10. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\frac{1}{-i}$$

Solution: hint: the bottom is $0-1i$

11. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^4$$

Solution: $i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$

12. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{14}$$

Solution:

$$\begin{aligned}i^{14} &= i^{12}\cdot i^2 && \text{(just add exponents)}\\ &= \left(i^4\right)^3\cdot i^2 && \text{(setting up to use fact } i^4=1\text{)}\\ &= (1)^3\cdot i^2 && \text{(yipei-kae-yeh...)}\\ &= i^2\\ &= -1\end{aligned}$$

13. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{25}$$

Solution:

$$\begin{aligned} i^{25} &= \left(i^{24}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^4\right)^6 \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^6 \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

14. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$\begin{aligned} i^{-3} &= i^{-3} \cdot 1 \\ &= i^{-3} \cdot i^4 && \text{(see previous problem why } i^4 = 1\text{)} \\ &= i^1 && \text{(just add exponents)} \\ &= i && \text{(note: there are many other ways to do this problem)} \end{aligned}$$

15. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{25}$$

Solution:

$$\begin{aligned} i^{25} &= \left(i^{24}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^4\right)^6 \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^6 \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

16. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-5}$$

Solution:

$$
\begin{aligned}
i^{-5} &= \left(i^{-8}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= \left(i^4\right)^{-2} \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= (1)^{-2} \cdot i && \text{(used } i^4 = 1\text{)} \\
&= i && \text{(ez as sundays)}
\end{aligned}
$$

17. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-7}$$

Solution:

$$
\begin{aligned}
i^{-7} &= \left(i^{-8}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= \left(i^4\right)^{-2} \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= (1)^{-2} \cdot i && \text{(used } i^4 = 1\text{)} \\
&= i && \text{(ez as sundays)}
\end{aligned}
$$

18. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$
\begin{aligned}
i^{-3} &= \left(i^{-4}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= \left(i^4\right)^{-1} \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= (1)^{-1} \cdot i && \text{(used } i^4 = 1\text{)} \\
&= i && \text{(ez as sundays)}
\end{aligned}
$$

19. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-5}$$

Solution:

$$
\begin{aligned}
i^{-5} &= \left(i^{-8}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= \left(i^4\right)^{-2} \cdot i^1 && \text{(preparing to use } i^4 = 1\text{)} \\
&= (1)^{-2} \cdot i && \text{(used } i^4 = 1\text{)} \\
&= i && \text{(ez as sundays)}
\end{aligned}
$$

20. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^2$$

Solution:

$$\begin{aligned} i^2 &= \left(i^0\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= \left(i^4\right)^0 \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= (1)^0 \cdot i && \text{(used } i^4 = 1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

21. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$\begin{aligned} i^{-3} &= \left(i^{-4}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= \left(i^4\right)^{-1} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= (1)^{-1} \cdot i && \text{(used } i^4 = 1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

22. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-3}$$

Solution:

$$\begin{aligned} i^{-3} &= \left(i^{-4}\right) \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= \left(i^4\right)^{-1} \cdot i^1 && \text{(preparing to use } i^4 = 1) \\ &= (1)^{-1} \cdot i && \text{(used } i^4 = 1) \\ &= i && \text{(ez as sundays)} \end{aligned}$$

23. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{11}$$

Solution:

$$\begin{aligned} i^{11} &= \left(i^{8}\right) \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^{4}\right)^{2} \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^{2} \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

24. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-6}$$

Solution:

$$\begin{aligned} i^{-6} &= \left(i^{-8}\right) \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^{4}\right)^{-2} \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^{-2} \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

25. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{33}$$

Solution:

$$\begin{aligned} i^{33} &= \left(i^{32}\right) \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^{4}\right)^{8} \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^{8} \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

26. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$i^{-150}$$

Solution:

$$\begin{aligned} i^{-150} &= \left(i^{-152}\right) \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= \left(i^{4}\right)^{-38} \cdot i^{1} && \text{(preparing to use } i^4 = 1\text{)} \\ &= (1)^{-38} \cdot i && \text{(used } i^4 = 1\text{)} \\ &= i && \text{(ez as sundays)} \end{aligned}$$

27. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^2$$

Solution:

$$\begin{aligned}
\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^2 &= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) \\
&= \frac{1}{2} + \frac{1}{2}i + \frac{1}{2}i + \frac{1}{2}i^2 && \text{(FOIL)} \\
&= \frac{1}{2} + \frac{1}{2}i + \frac{1}{2}i - \frac{1}{2} && \text{(used } i^2 = -1\text{)} \\
&= i
\end{aligned}$$

28. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2$$

Solution:

$$\begin{aligned}
\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2 &= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \\
&= \frac{3}{4} + \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i + \frac{1}{4}i^2 && \text{(FOIL)} \\
&= \frac{3}{4} + \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i - \frac{1}{4} && \text{(used } i^2 = -1\text{)} \\
&= \frac{2}{4} + \frac{2\sqrt{3}}{4}i \\
&= \frac{1}{2} + \frac{\sqrt{3}}{2}i
\end{aligned}$$

29. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3$$

Solution: we will use the result from the previous problem:

$$\begin{aligned}
\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2 &= \frac{1}{2}+\frac{\sqrt{3}}{2}i && \text{(prev. problem)}\\
\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) &= \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) && \text{(mult both sides by } \left(\tfrac{\sqrt{3}}{2}+\tfrac{1}{2}i\right)\text{)}\\
\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3 &= \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) && \text{(simplify left side)}\\
&= \frac{\sqrt{3}}{4}+\frac{3}{4}i+\frac{1}{4}i+\frac{\sqrt{3}}{4}i^2 && \text{(FOIL)}\\
&= \frac{\sqrt{3}}{4}+\frac{3}{4}i+\frac{1}{4}i-\frac{\sqrt{3}}{4} && \text{(use } i^2=-1\text{)}\\
&= i && \text{(yipi-kae-yeh)}
\end{aligned}$$

30. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^6$$

Solution: just for kicks.. and for the simplicity of writing, let us call this number $\frac{\sqrt{3}}{2}+\frac{1}{2}i = A$. On the previous problem we demonstrated that $A^3 = i$, we will use this fact freely and without inhibitions ... now..

$$\begin{aligned}
A^6 &= \left(A^3\right)^2 && \text{(getting ready to use } A^3=i\text{)}\\
&= (i)^2 && \text{(used } A^3=i\text{)}\\
&= -1 && \text{(ez as Sundays...)}
\end{aligned}$$

31. **Compute w/ Complex Numbers** Calculate and write in standard form.

$$\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^{30}$$

Solution: just for kicks.. and for the simplicity of writing, let us call this number $\frac{\sqrt{3}}{2}+\frac{1}{2}i = A$. On the previous problem we demonstrated that $A^3 = i$, we will use this fact freely and without inhibitions ... now..

$$\begin{aligned}
A^6 &= \left(A^3\right)^{10} && \text{(getting ready to use } A^3=i\text{)}\\
&= (i)^{10} && \text{(used } A^3=i\text{)}\\
&= -1 && \text{(ez as Sundays...)}
\end{aligned}$$

32. **Inventing Numbers** The natural numbers are in many ways natural. In some way, all other numbers are unnatural byproducts of human imagination. Which number was *invented* just to solve the following equation?

$$3 + x = 3$$

> **Solution:** 0 was innvented

33. **Inventing Numbers** Which type of numbers were *invented* to solve the following equation?

$$3 + x = 0$$

> **Solution:** negative numbers were innvented

34. **Inventing Numbers** Which type of numbers were *invented* to solve the following equation?

$$3x = 1$$

> **Solution:** rational numbers were innvented

35. **Inventing Numbers** Which type of numbers were *invented* to solve the following type of equation?

$$x^2 = 3$$

> **Solution:** square roots of numbers were innvented

36. **Inventing Numbers** Contemplate the idea of a world of numbers of the form $a + b\sqrt{3}$ where a, b are rational numbers.
    (a) add $\frac{2}{3} + 5\sqrt{3} + \frac{7}{3} + \frac{3}{5}\sqrt{3}$
    (b) multiply $\left(\frac{2}{3} + 5\sqrt{3}\right)\left(\frac{7}{3} + \frac{3}{5}\sqrt{3}\right)$
    (c) does $\frac{2}{3} + 5\sqrt{3}$ have a multiplicative inverse of the form $a + b\sqrt{3}$ where a, b are rational.

37. Is $\sqrt{i}$ a complex number? if so can you write it in standard form?

> **Solution:** yes.. the answer is on prob #35