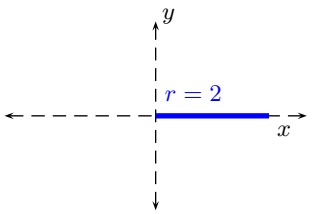
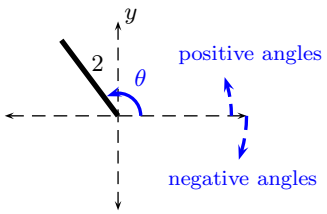
Reference Triangles

Every angle has a reference triangle. The recipe is as follows:

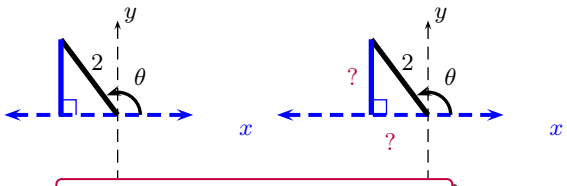
STEP 1. Choose any $+r$



STEP 2. Rotate θ

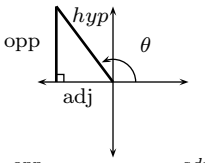


STEP 3. Draw a Perp to x-axis **STEP 4**. Label sides/signs



Def: of THE Trig Functions

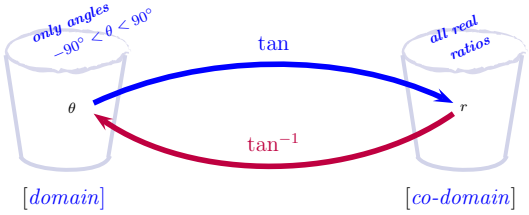
angle θ → ref. triangle for θ → ratio of sides



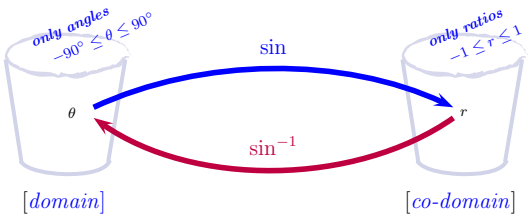
$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

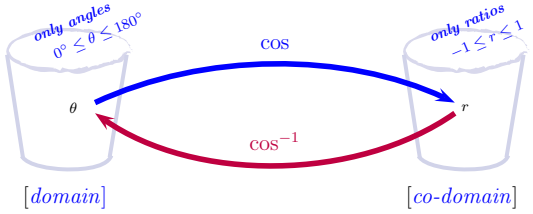
Def: Inverse Tangent Function



Def: Inverse Sine Function

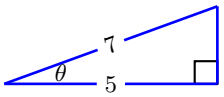


Def: Inverse Cosine Function



Example:

Find the angle θ



In this case, we know the adjacent and the hypothenuse sides. The function describing such ratio is the cosine, thus

$$\cos\theta = \frac{5}{7}$$

Since this is not a famous ratio, we allow ourselves use of a calculator to estimate the sought angle.

$$\theta = \cos^{-1}\left(\frac{5}{7}\right)$$

$$\approx 0.775$$

(in radians [see calculator mode]... OR...)

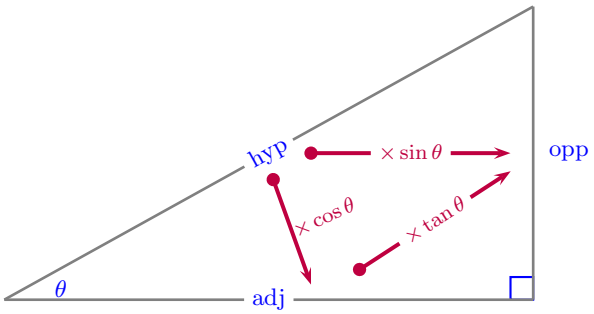
$$\approx 44.415°$$

(in degrees [see calculator mode])

... it should be noted that we will revisit the equation $\cos\theta = \frac{5}{7}$ under a different context, where we will solve it completely, not limited to the domain and codomain of the arccos function.

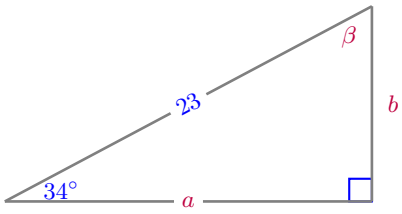
LOP Diagram:

- The essential concept 1: *once the ratios are known, one side is enough to determine the other sides*
- The essential concept 2: *For each angle, the ratios are described by the trig functions where defined.*



Solving Euclidean Right Triangles

Solve the triangle [may not be drawn to scale assume typical units for length]:



Solution:
Notice, this proposition is typical, three items are given; the right angle, the 34 angle, and the hypothenuse side, while three items are missing; sides a, b, and angle β.
It should be noted that the angle β is relatively easy to determine $\beta + 34° = 90°$ Therefore, $\beta = 56°$.
Now, we solve for b, the side opposite of the 34° angle. Since we know the hypothenuse is 23 units, and we want to know the sine function describes this ratio, $\sin 34° \approx 0.559$, similarly to solve for a we use the cosine ratio, $\cos 34° \approx 0.829$. Thus, we illustrate the ratios on the triangle: