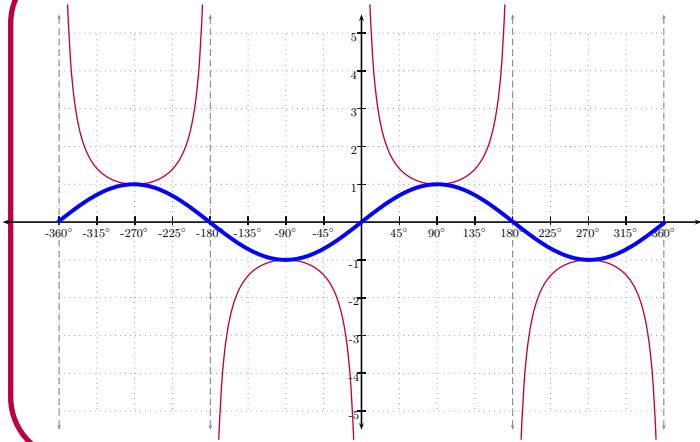


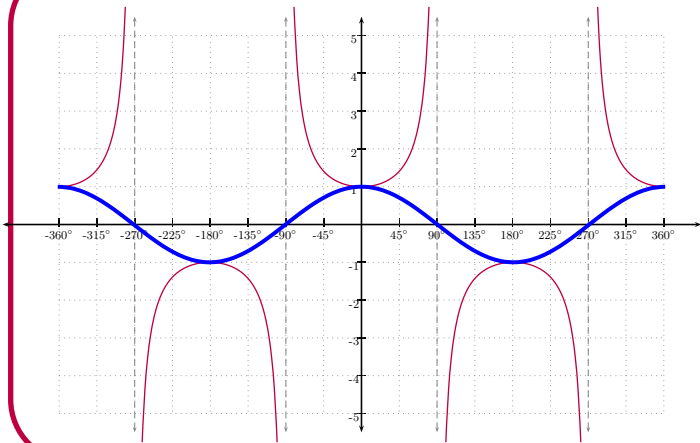
Graphs:  $\sin x$  &  $\csc x$

The graphs of  $y = \sin x$  and the graph of  $y = \csc x$



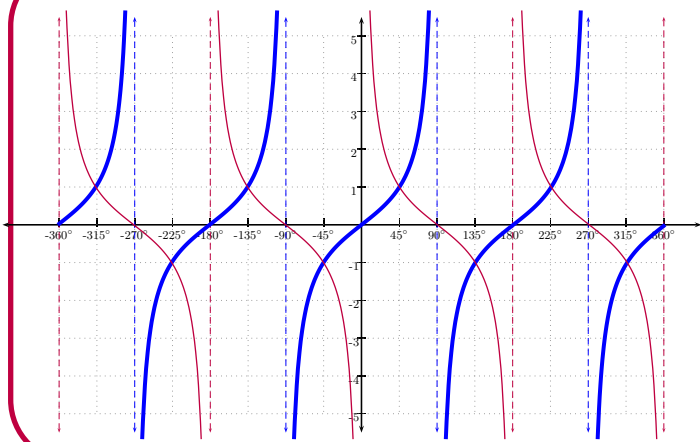
Graphs:  $\cos x$  &  $\sec x$

The graphs of  $y = \cos x$  and the graph of  $y = \sec x$



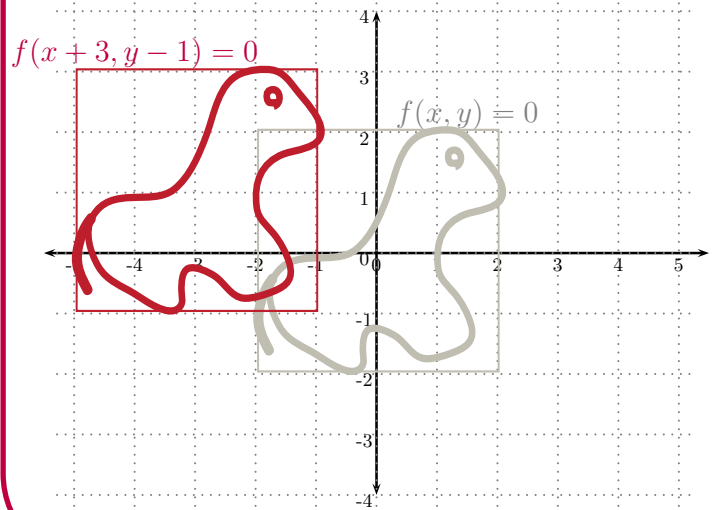
Graphs:  $\tan x$  &  $\cot x$

The graphs of  $y = \tan x$  and the graph of  $y = \cot x$



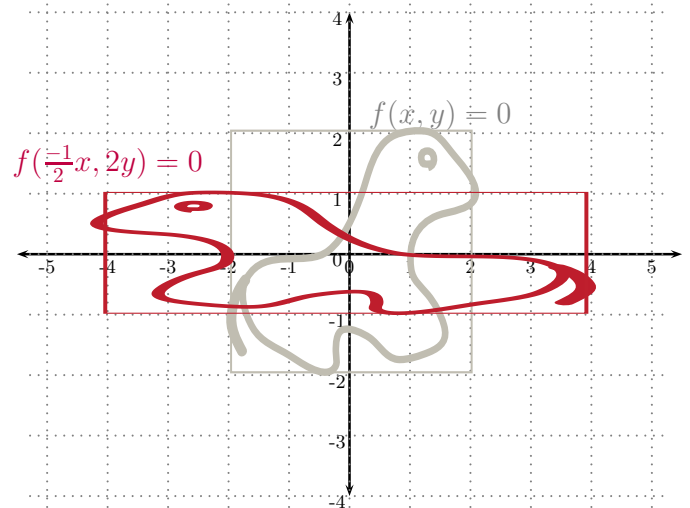
Graphs: Shifting Principle

Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  and  $h, k \in \mathbb{R}^+$ . The graph of  $f(x - h, y - k) = 0$  is the same as the shifted graph of  $f(x, y) = 0$ ,  $h$  units to the right and  $k$  units up. Similarly, the graph of  $f(x + h, y + k) = 0$  is the same as the shifted graph of  $f(x, y) = 0$ ,  $h$  units to the left and  $k$  units down.



Graphs: Scale/Reflect Principle

Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  and  $h, k \in \mathbb{R}^+$ . The graph of  $f(hx, ky) = 0$  is the same as the compressed/stretched graph of  $f(x, y) = 0$ , by a factor of  $\frac{1}{h}$  horizontally, and  $\frac{1}{k}$  vertically. If  $h$  is negative, the graph is reflected about the  $y$ -axis. Similarly if  $k$  is negative.



Graphs: Scale & Shift

Use the Scale, & Shift principles to graph

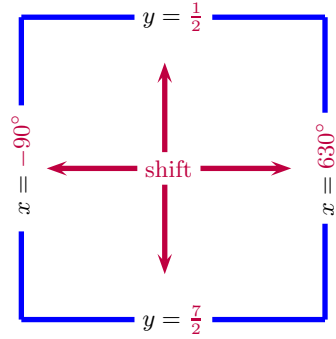
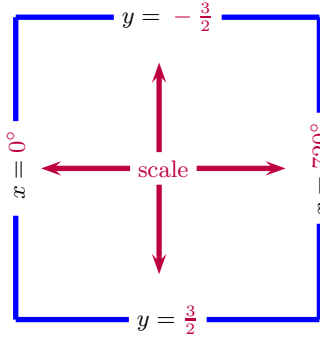
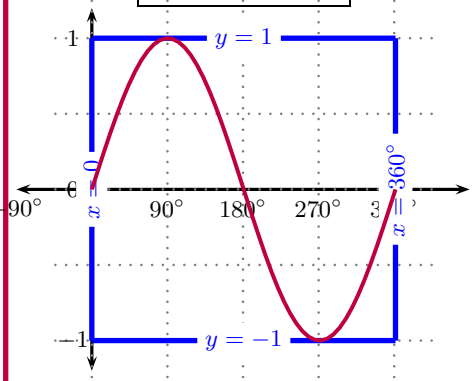
$$-\frac{2}{3}(y-2) = \sin\left[\frac{1}{2}(x+90^\circ)\right]$$

**Solution:**

**basic:**  $y = \sin(x)$

**scale:**  $-\frac{2}{3}y = \sin\left(\frac{1}{2}x\right)$

**shift:**  $-\frac{2}{3}(y-2) = \sin\frac{1}{2}(x+90^\circ)$



Final [one period of] graph for

$$-\frac{2}{3}(y-2) = \sin\left[\frac{1}{2}(x+90^\circ)\right]$$

