math hands

Famous ID's

Definition Identities

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}$$
$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Co-Function Identities

$$\sin\theta = \cos(90^\circ - \theta) \qquad \tan\theta = \cot(90^\circ - \theta)$$
$$\sec\theta = \csc(90^\circ - \theta) \qquad \sin\theta = \cos(\theta - 90^\circ)$$
$$-\cos\theta = \sin(\theta - 90^\circ) \qquad \cos\theta = \sin(90^\circ - \theta)$$

Even & Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$
$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Pythagoras Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$
$$\sin^2\theta = 1 - \cos^2\theta \qquad \cos^2\theta = 1 - \sin^2\theta$$
$$\tan^2\theta = \sec^2\theta - 1 \qquad \cot^2\theta + 1 = \csc^2\theta$$
$$\cot^2\theta = \csc^2\theta - 1 \qquad \sin\theta = \pm\sqrt{1 - \cos^2\theta}$$
$$\tan\theta = \pm\sqrt{\sec^2\theta - 1} \qquad \sec\theta = \pm\sqrt{\tan^2\theta + 1}$$

General Strategies to Prove Identities

1. **Work on sides independently.**
2. **Tweak a known Identity.**
3. **Look at Graphs.**
4. **Start with something amazingly creative and brilliant!**

Sum-Angle Identities

the M.O.T.A. $\longmapsto$

$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$
$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

Double-Angle Identities

$$\cos(2a) = \cos^2 a - \sin^2 a \qquad \cos(2a) = 2\cos^2 a - 1$$
$$\cos(2a) = 1 - 2\sin^2 a \qquad \sin(2a) = 2\sin a\cos a$$
$$\tan(2a) = \frac{2\tan a}{1 - \tan^2 a} \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

Product-to-Sums Identities

$$\sin a\sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$
$$\cos a\cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$
$$\sin a\cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

Sums-to-Products Identities

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin a - \sin b = 2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$
$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Euler's Identity

$$e^{i\theta} = \cos\theta + i\sin\theta$$