1. Solve

$$|2x + 3| = 5$$

2. Solve

$$|-3x + 1| = 2x$$

3. Solve

$$|2x + 3| = -x + 1$$

4. Solve

$$|2x + 3| = x + 1$$

5. Solve

$$|2x + 4| = 1$$

6. solve $|x+3|+2|x|=5$

7. Solve

$$|-3x+1|=2$$

8. Solve

$$|x+3|=5$$

9. Solve

$$-4|2x-3|+5x=2$$

10. Solve

$$|-3x+1|=2x+3$$

11. Prove AVT (if you can't prove it, just try to prove it for at least 15 hours, not consecutive hours)

12. Solve

$$|-3x+1| = -2x+3$$

13. Solve

$$|5x-1| = -x-1$$

14. Solve

$$|2x-4| = 2$$

15. Solve

$$2|5x-1| + x = 1$$

THE IDEA

In this section we consider equations containing Absolute Value Expressions. The general strategy will be to isolate the term containing a the absolute value, then use the definition of absolute value to eliminate it from the equation. This generally turns the equation into two equations less the absolute value expression that was isolated. From that point, one can use the skills and tools of polynomial equations. If the equation has more than one term with an absolute value, one can annihilate *one absolute value at a time* by isolating *it*, then using the definition of absolute value, then repeating the process for other absolute value expressions that were not isolated and annihilated. As before, solutions should be checked at the end.

THE DEFINITION of Absolute Value

Recall, we define the absolute value function with domain $\mathbb{R}$ and codomain $\mathbb{R}$ and

$$abs(x) = \begin{cases} x & \text{if } x \in \mathbb{R}^+, \\ -x & \text{otherwise} \end{cases}$$

THE THEOREM Absolute Value Theorem [AVT]

For any real quantity a,

$$\text{IF } |x| = a \text{ THEN } x = a \text{ or } x = -a$$

The reader is invited to prove this theorem, and such proof may be found to be an amusing mental exercise.

We proceed to show how the theorem helps solve equations with absolute values in them. The principal idea is that it take an absolute value expression [that has been isolated on one side of equation] and abolishes the absolute value by breaking up the equation into two equations without *that* absolute value.

THE EXAMPLE Solving an equation w/Absolute Value

Solve

$$|2x - 4| = 2$$

Solution:

$$\begin{array}{rcll} \text{if} & |2x-4| = 2 & & \text{(is given, then)} \\ 2x - 4 = 2 & \text{or} & 2x - 4 = -2 & \text{(AVT)} \\ 2x = 6 & \text{or} & 2x = 2 & \text{(BI)} \\ x = 3 & \text{or} & x = 1 & \text{(BI)} \end{array}$$

We can double check the solutions, to see if each works. They each work in this case.

THE CAUTION Solving an equation w/Absolute Value

Solve

$$|2x - 4| = -10$$

Solution:

$$
\begin{aligned}
&\text{if} \quad |2x-4| = -10 && \text{(is given, then)}\\
2x-4 = -10 \quad &\text{or} \quad 2x-4 = -(-10) && \text{(AVT)}\\
2x = -6 \quad &\text{or} \quad 2x = 10 && \text{(BI)}\\
x = -3 \quad &\text{or} \quad x = 5 && \text{(BI)}
\end{aligned}
$$

We can double check the solutions, to see if each works. Neither one works in this case. Of course, it was easy to see that the equation was doomed from the beginning because the absolute value of any real number being equal to -10 was absurd from the start. Note this does not show that the AVT theorem is imperfect, it simply shows that the IF part of the statement was false from the start. That is why it is good practice to check the potential solutions obtained. Moreover, there may be times when it is not so easy to determine wether or not the equation is doomed from the start. Consider the following example.

THE NEXT EXAMPLE Solving an equation w/Absolute Value

Solve

$$|2x-4| = -3x+1$$

Solution:

$$
\begin{aligned}
&\text{if} \quad |2x-4| = -3x+1 && \text{(is given, then)}\\
2x-4 = -3x+1 \quad &\text{or} \quad 2x-4 = -(-3x+1) && \text{(AVT)}\\
5x = 5 \quad &\text{or} \quad 2x-4 = 3x-1 && \text{(BI)}\\
5x = 5 \quad &\text{or} \quad -x = 3 && \text{(BI)}\\
x = 1 \quad &\text{or} \quad x = -3 && \text{(BI)}
\end{aligned}
$$

We check each potential solution, 1 does NOT work, but $x = -3$ does work!! note, it is not clear, from the start, wether or not this equation is doomed, since '$-3x+1$' could be just about anything depending on the value of x.