math hands
Big & Famous Ideas

Famous Limits

$$\lim_{x\to 0}\frac{\sin(x)}{x}=1 \qquad \lim_{x\to 0}\frac{1-\cos(x)}{x}=0$$
$$\lim_{x\to\infty}(.998)^x=0 \qquad \lim_{x\to\infty}(1.003)^x=\infty$$
$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e \qquad \lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x=e^k$$

Famous Limit Ideas

$$\lim_{x\to c}[k]=k \tag{Lk}$$
$$\lim_{x\to c}[x]=c \tag{Lx}$$
$$\lim_{x\to c}[kf(x)]=k\lim_{x\to c}[f(x)) \tag{Pk*}$$
$$\lim_{x\to c}[f(x)+g(x)]=\lim_{x\to c}[f(x))+\lim_{x\to c}(g(x)] \tag{LS=SL*}$$
$$\lim_{x\to c}[f(x)\cdot g(x)]=\lim_{x\to c}[f(x))\cdot\lim_{x\to c}(g(x)] \tag{LP=PL*}$$
$$\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to c}[f(x))}{\lim_{x\to c}(g(x)]} \tag{LQ=QL*}$$
$$\lim_{x\to c}[f(g(x))]=f\left[\lim_{x\to c}g(x)\right] \tag{limTcont**}$$
$$\lim_{x\to c}f(x)=L \iff \tag{THE definition}$$
$$\forall\epsilon>0\exists\delta>0\colon 0<|x-c|<\delta \implies |f(x)-L|<\epsilon$$

*when each exists, is finite, and denominators not zero, k and c are real numbers **when $\lim_{x\to c}g(x)=L$ each exists, is finite, and $f(x)$ is continuous at $x=L$

Famous Indeterminate Forms

$$0^0 \qquad \infty^0$$
$$1^\infty \qquad \infty\cdot 0$$
$$\frac{0}{0} \qquad \frac{\infty}{\infty}$$
$$\infty-\infty \qquad \sin(\infty) \quad (*)$$

*last one is not that famous

Big Deal on Continuous

- $$\lim_{x\to c}[f(g(x))]=f\left[\lim_{x\to c}g(x)\right]$$
- if f and g are continuous at some point $x=c$ then

  $f+g,\quad f\cdot g,\quad f/g, kf+cg,$ are also continuous
- Intermediate Value Theorem
- EVT $\implies$ Rolles's Thm $\implies$ MVT $\implies$ FTC

Famous Derivatives

$$[k]'=0 \qquad [x]'=1$$
$$\left[x^k\right]'=kx^{k-1} \qquad [\ln(x)]'=\frac{1}{x} \quad *$$
$$[e^x]'=e^x \qquad [stuff]'=\left[e^{\ln(stuff)}\right]' \quad **$$
$$[\sin(x)]'=\cos(x) \qquad [\cos(x)]'=-\sin(x)$$
$$[\tan(x)]'=\sec^2(x) \qquad [\cot(x)]'=-\csc^2(x)$$
$$[\sec(x)]'=\sec(x)\tan(x) \qquad [\csc(x)]'=-\csc(x)\cot(x)$$
$$[\sinh(x)]'=\cosh(x) \qquad [\cosh(x)]'=\sinh(x)$$
$$[\arctan(x)]'=\frac{1}{x^2+1} \qquad [\text{arccot}(x)]'=\frac{-1}{x^2+1}$$
$$[\arcsin(x)]'=\frac{1}{\sqrt{1-x^2}} \qquad [\arccos(x)]'=\frac{-1}{\sqrt{1-x^2}}$$

* need $x>0$
** $stuff$ can be any real function $g(x)>0$

Derivatives chain rule edition

$$\frac{d}{dx}\left[(stuff)^k\right]=k(stuff)^{k-1}\cdot\frac{d}{dx}[stuff] \tag{*}$$
$$\frac{d}{dx}[\ln(stuff)]=\frac{1}{stuff}\cdot\frac{d}{dx}[stuff] \tag{**}$$
$$\frac{d}{dx}\left[e^{stuff}\right]=e^{stuff}\cdot\frac{d}{dx}[stuff] \tag{*}$$
$$\frac{d}{dx}[\sin(stuff)]=\cos(stuff)\cdot\frac{d}{dx}[stuff] \tag{*}$$
$$\frac{d}{dx}[\arctan(stuff)]=\frac{1}{(stuff)^2+1}\cdot\frac{d}{dx}[stuff] \tag{*}$$
$$\frac{d}{dx}[\sec(stuff)]=\sec(stuff)\tan(stuff)\cdot\frac{d}{dx}[stuff] \tag{*}$$

* $stuff$ can be any real function $g(x)$, when each derivative exists and is finite
** $stuff$ can be any real function $g(x)>0$, when each derivative exists and is finite

famous derivative ideas

$$(f+g)'=f'+g' \qquad (kf)'=k(f)' \qquad [f(g)]'=f'(g)g'$$
$$(f\cdot g)'=f'\cdot g+f\cdot g' \qquad \left(\frac{f}{g}\right)'=\frac{f'\cdot g-f\cdot g'}{g^2}$$
$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c} \qquad f'(c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

if each derivative exists and is finite blah blah..

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Definition Identities

$\sin\theta = \frac{opp}{hyp}$ $\quad$ $\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$ $\quad$ $\cot\theta = \frac{adj}{opp}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$

Co-Function Identities

$\sin\theta = \cos(90^\circ - \theta)$ $\quad$ $\tan\theta = \cot(90^\circ - \theta)$

$\sec\theta = \csc(90^\circ - \theta)$ $\quad$ $\sin\theta = \cos(\theta - 90^\circ)$

$-\cos\theta = \sin(\theta - 90^\circ)$ $\quad$ $\cos\theta = \sin(90^\circ - \theta)$

Even & Odd Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$

$\csc(-\theta) = -\csc\theta$ $\quad$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Pythagoras Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$ $\quad$ $\cos^2\theta = 1 - \sin^2\theta$

$\tan^2\theta = \sec^2\theta - 1$ $\quad$ $\cot^2\theta + 1 = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$ $\quad$ $\sin\theta = \pm\sqrt{1 - \cos^2\theta}$

$\tan\theta = \pm\sqrt{\sec^2\theta - 1}$ $\quad$ $\sec\theta = \pm\sqrt{\tan^2\theta + 1}$

General Strategies to Prove Identities

1. **Work on sides independently.**
2. **Tweak a known Identity.**
3. **Look at Graphs.**
4. **Start with something amazingly creative and brilliant!**

Sum-Angle Identities

the M.O.T.A. $\longmapsto$

$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$
$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$
$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$
$$\sin(a-b) = \sin a\cos b - \cos a\sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$$

Double-Angle Identities

$\cos(2a) = \cos^2 a - \sin^2 a$ $\quad$ $\cos(2a) = 2\cos^2 a - 1$

$\cos(2a) = 1 - 2\sin^2 a$ $\quad$ $\sin(2a) = 2\sin a\cos a$

$\tan(2a) = \frac{2\tan a}{1 - \tan^2 a}$ $\quad$ $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

Product-to-Sums Identities

$$\sin a\sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$
$$\cos a\cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$
$$\sin a\cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

Sums-to-Products Identities

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin a - \sin b = 2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$
$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

Euler's Identity

$$e^{i\theta} = \cos\theta + i\sin\theta$$