math hands

Derivatives Good for...

Sir Newton

- [good for..] solving equations
- [when...] for given a differentiable function $f(x)$, roughly speaking, when one can find a point, x_1 near a root, c of $f(x)$, then most of the time, repeated application of newton will take us closer to the solution, "c"
- [how...]

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

L'Hopital

- [good for..] L'Hopital is an excellent option to find limits.
- [when...]

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \frac{\pm\infty}{\infty} \qquad \text{(L'H form, and..)}$$

$$\lim_{x\to c} \frac{f'(x)}{g'(x)} \quad \text{exists} \qquad \text{(then..L'H applies)}$$

- [how..]

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{2x}{1} = 6 \qquad \text{(L'H...humm fancy huh..)}$$

Linear Approximations

- [good for..] Approximating some values of some functions.
- [when...] When $f(c)$ is difficult to evaluate, but the equation of the line tangent to $f(x)$ near c is easy(-ier?).
- [how..] example: approximate $\sqrt[3]{132}$

First note $\sqrt[3]{125}$ is easy to compute and near $\sqrt[3]{132}$, so we find the equation of the tangent line to $f(x) = x^{\frac{1}{3}} = \sqrt[3]{x}$:

$$f(x) = x^{\frac{1}{3}} \qquad \text{(given)}$$

$$f(125) = 5 \qquad \text{(we'll need it..)}$$

$$f'(x) = \frac{1}{3}(x)^{\frac{-2}{3}} \qquad \text{(will need it.)}$$

$$f'(125) = \frac{1}{3}(125)^{\frac{-2}{3}} = \frac{1}{75} \qquad \text{(will need it.)}$$

$$p(x) = f(125) + f'(125)(x-125) \qquad \text{(standard eq of tan line)}$$

$$p(x) = 5 + \frac{1}{75}(x-125) \qquad \text{(plug in)}$$

$$p(132) = 5 + \frac{1}{75}(132-125) = \frac{382}{75} \qquad \text{(plug in)}$$

thus $\sqrt[3]{132} \approx \frac{382}{75}$

Related Rates

- [good for..] describing rate of change of one quantity, A when the rate of change of another quantity B is known. The relationship between A and B must also be known.
- [how...] example: suppose we know $A^2 + 5B = \ln(B)$ then simply take $\frac{d}{dt}[]$ on each side, and plug in known quantities..and solve for wanted rate...

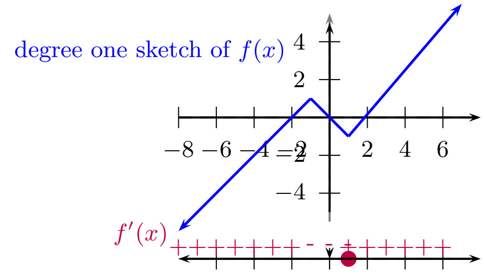
$$A^2 + 5B = \ln(B) \qquad \text{(given)}$$

$$\frac{d}{dt}\left[A^2 + 5B\right] = \frac{d}{dt}\left[\ln(B)\right] \qquad \text{(take } \tfrac{d}{dt}[]\text{)}$$

$$2A\frac{dA}{dt} + 5\frac{dB}{dt} = \frac{1}{B}\frac{dB}{dt} \qquad \text{(now just plug in and solve)}$$

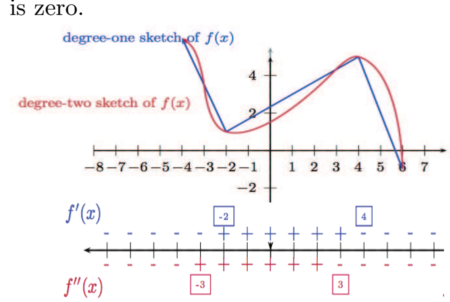
Curve Sketching

- [good for..] sketching curves
- [when...] for given function $f(x)$, when one can study the sign of the first derivative of a function $f'(x)$ one can tell where the function is increasing or decreasing since $f'(c) > 0$ implies f increasing near c and $f'(c) < 0$ implies f decreasing near c Key Idea intervals where derivative is positive and negative are separated by critical points, points where the derivative DNE or is zero.



Curve Sketching II

- [good for..] sketching curves
- [when...] for given function $f(x)$, when one can study the sign of the first & second derivatives of a function one can tell concavity of $f(x)$ since $f''(c) > 0$ implies concave up near c and $f''(c) < 0$ implies f concave down near c Key Idea intervals where second derivative is positive and negative are separated by critical points, points where the second derivative DNE or is zero.



Optimization

- [good for..] finding max or min value of a function
- [key idea...] according to *Fermat Theorem* if a differentiable function $f(x)$ attains a local max/min at c then $f'(c) = 0$ thus to find local max/min we look to the places where f is not differentiable or has derivative zero.
- [general strategy to solve applied problems]

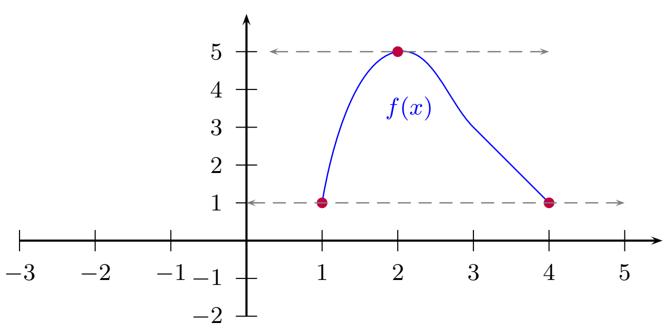
  1. *read:* the problem until you can identify which function is to be optimized, call it f.
  2. *formula:* determine a formula for f.
  3. *variables:* if the found formula for f contains more than one variable, eliminate all but one.
  4. *seek critical pts:* critical points (and endpoints) are candidates for local/absolute max/min
  5. *test each critical point* can use second derivative, ex. $f''(c) > 0$ means concave up, means local min.

Rolle's Theorem

- [says...] if f differentiable over $[a, b]$ and $a < b$ then.... IF $f(a) = f(b)$ there exists a point $c \in (a, b)$ such that

$$f'(c) = 0$$

- [key picture...]



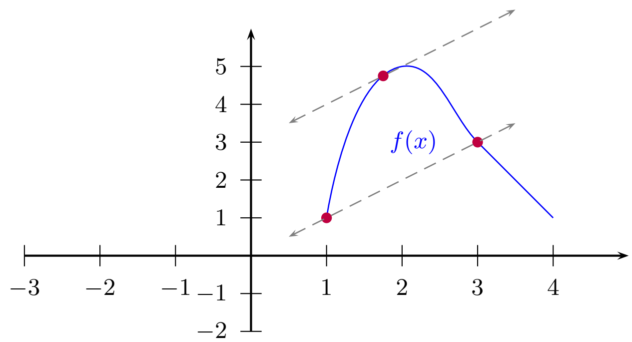
- [importance...] it helps prove the *Mean Value Theorem*

Mean Value Theorem

- [says...] if f differentiable over $[a, b]$ and $a < b$ then.... there exists a point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- [key picture...]



- [important...] by itself, like a very forgettable midwife, but it delivers BIGtime *THE Fundamental Theorem of Calculus*
- [UBER-important edition] if $F'(x) = f(x)$ then

$$f(c)(b - a) = F(b) - F(a)$$