math hands

Areas & Integrals

famous sums

$$\sum_{k=1}^{p} c = pc \qquad \sum_{k=1}^{p} k = \frac{p(p+1)}{2}$$

$$\sum_{k=1}^{p} k^2 = \frac{p(p+1)(2p+1)}{6} \qquad \sum_{k=1}^{p} k^3 = \frac{p^2(p+1)^2}{4}$$

$$\sum_{k=0}^{p} r^k = \frac{r^{p+1}-1}{r-1}$$

finite sums: properties

$$\sum_{k=1}^{p}(a_k+b_k) = \sum_{k=1}^{p} a_k + \sum_{k=1}^{p} b_k \qquad \sum_{k=1}^{p} c\cdot a_k = c\sum_{k=1}^{p} a_k$$

$$\sum_{k=1}^{p} a_k = \sum_{k=1}^{m} a_k + \sum_{k=m}^{p} a_k \qquad \sum_{k=m}^{p} a_k = \sum_{k=m-n}^{p-n} a_{k+n}$$

fundamental theorem of calculus

- version A if $f(t)$ is an integrable real function on $[a,b]$ and assume $G(x)$ is an antiderivative of $f(x)$ on $[a,b]$, then

$$\int_a^b f(t)dt = F(b) - F(a)$$

- version B if $f(t)$ is a continuous real function on $[a,b]$ then, $f(x)$ has an antiderivative that is continuous and differentiable on the interval (a,b), namely, $F(x)$ defined by

$$F(x) = \int_a^x f(t)dt$$

finite integral properties

$$\int_a^a f(t)dt = 0$$

$$\int_a^b f(t)dt = -\int_b^a f(t)dt$$

$$\int_a^b kf(t)dt = k\int_a^b f(t)dt$$

$$\int_a^c f(t)dt = \int_a^b f(t)dt + \int_b^c f(t)dt$$

$$\int_a^c f(t)dt = \int_{a-n}^{c-n} f(t+n)dt$$

basic integrals

$$\int dt = t + c$$

$$\int t^m dt = \frac{1}{m+1}t^{m+1} + c$$

$$\int \frac{1}{t}dt = \ln|t| + c$$

$$\int e^t dt = e^t + c$$

$$\int \cos(t)dt = \sin(t) + c$$

$$\int \sin(t)dt = -\cos(t) + c$$

$$\int \cosh(t)dt = \sinh(t) + c$$

$$\int \sinh(t)dt = \cosh(t) + c$$

$$\int \sec^2(t)dt = \tan(t) + c$$

$$\int \csc^2(t)dt = -\cot(t) + c$$

$$\int \csc(t)\cot(t)dt = -\csc(t) + c$$

$$\int \sec(t)\tan(t)dt = \sec(t) + c$$

non-basic memorable integrals

$$\int \tan(t)dt = -\ln|\cos(t)| + c$$

$$\int \sec(t)dt = \ln|\sec(t) + \tan(t)| + c$$

$$\int \csc(t)dt = \ln|\csc(t) - \cot(t)| + c$$

$$\int \sin^2(t)dt = \frac{1}{2}t - \frac{1}{4}\sin(2t) + c$$

$$\int \cos^2(t)dt = \frac{1}{2}t + \frac{1}{4}\sin(2t) + c$$

$$\int \frac{1}{\sqrt{1-t^2}}dt = \arcsin(t) + c$$ (maybe basic..)

$$\int \frac{1}{t\sqrt{t^2-1}}dt = \text{arcsec}(t) + c$$ (maybe basic..)

$$\int \frac{1}{t^2+1}dt = \arctan(t) + c$$ (maybe basic..)

$$\int \frac{-1}{t^2+1}dt = \text{arccot}(t) + c$$ (maybe basic..)

$$\int \ln(t)dt = t\ln(t) - t + c$$ (assume $t > 0$)